CS 161	**Problem Set 1**
Winter 2021	**Due:** Wed, Jan 20 at 11:59 pm PST

Style guide and expectations: Please see the "Homework" part of the "Resources" section on the webpage for guidance on what we are looking for in homework solutions. We will grade according to these standards. You should cite all sources you used outside of the course material.

What we expect: Make sure to look at the "**We are expecting**" blocks below each problem to see what we will be grading for in each problem!

Exercises

We suggest you do these on your own. As with any homework problem, though, you may ask the course staff for help.

---

0. (1 pt.) Have you thoroughly read the course policies on the webpage?

[**We are expecting:** The answer "yes."]

1. (10 pt.) Using the definition of big-O, formally prove the following statements.

(a) $2\sqrt{n} + 6 = O(\sqrt{n})$.

(b) 10^n is **not** $O(2^n)$.

[**We are expecting:** For each part, a rigorous (but short) proof, using the definition of $O()$.]

2. (10 pt.) For each blank, indicate whether A_i is in O, Ω, or Θ of B_i. More than one space per row can be valid.

[**We are expecting:** All valid spaces in the table to be marked (checkmark, "x", etc.). No explanation is required.]

A	**B**	O	Ω	Θ
$\log^9 n$	$n^{0.9}$			
$2n^{10}$	2^n			
3^{3n}	3^{4n}			
$\ln n$	$\log n$			
$\log(n!)$	$\log(n^n)$			
$(5/4)^n$	$(4/5)^n$			
n^2	$4^{\log n}$			
$n^{0.1}$	$(0.1)^n$			
$\log\log n$	$\sqrt{\log n}$			
$n^{1/\log n}$	1			

Problems

You may talk with your fellow CS 161-ers about the problems. However:

- Try the problems on your own *before* collaborating.
- Write up your answers yourself, in your own words. You should never share your typed-up solutions with your collaborators.
- If you collaborated, list the names of the students you collaborated with at the beginning of each problem.

---

3. (10 pt.) The Fibonacci numbers are a famous sequence defined as

$$F_0 = 0 \qquad F_1 = 1 \qquad F_{n+2} = F_n + F_{n+1}$$

For example, the first few terms of the Fibonacci sequence are

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots$$

Show by induction that

(a) $F_n = O(3^n)$.

(b) $F_{2n} = \Omega(2^n)$.

[**We are expecting:** For each part, a formal proof by induction. Make sure to clearly label your inductive hypothesis, base case, inductive step, and conclusion.]

4. New friends. Each of n users spends some time on a social media site. For each $i = 1, ..., n$, user i enters the site at time a_i and leaves at time $b_i \geq a_i$. You are interested in the question: how many distinct pairs of users are on the site at the same time? (Here, the pair (i, j) is the same as the pair (j, i)).

Example: Suppose there are 5 users with the following entering and leaving times:

User	Enter Time	Leave Time
1	1	4
2	2	5
3	7	9
4	9	10
5	6	10

Then, the number of distinct pairs of users who are on the site at the same time is four: these pairs are (1, 2), (3, 4), (4, 5), (3, 5).

Note: If the Leave Time of one user is the same as the Enter Time of another, this counts as an overlap. For example, user 3's Leave Time is 9, and User 4's Enter Time is 9, and this counts as an overlap.

(a) **(3 pt.)** Given input $(a_1, b_1), (a_2, b_2), ..., (a_n, b_n)$ as above, there is a straightforward algorithm that takes about n^2 time to compute the number of pairs of users who are on the site at the same time. Give this algorithm and explain why it takes time about n^2.

[**We are expecting:** Pseudocode for your algorithm, a clear English description of what your algorithm is doing and why it is correct, and a brief runtime analysis. You do not need to prove that your algorithm is correct.]

(b) **(5 pt.)** Give an $O(n \log(n))$-time algorithm to do the same task and analyze its running time. (**Hint:** consider sorting relevant events by time).

[**We are expecting:** Pseudocode for your algorithm, a clear English description of what your algorithm is doing and why it is correct, and a brief runtime analysis. You do not need to prove that your algorithm is correct.]

5. **(1 pt.) Feedback: Prompt of the Week.** There's no "correct" answer here, and it is completely anonymous! Go to https://forms.gle/3PsgYHHRzvdHP8j36 and answer the prompt of the week. (You do not need to identify on the form to get the 1 point credit.)

   Did you fill out the poll?

   [**We are expecting:** The answer "yes."]