

Pre-lecture exercises will not be collected for credit. However, you will get more out of each lecture if you do them, and they will be referenced during lecture. We recommend **writing out** your answers to pre-lecture exercises before class. Pre-lecture exercises usually should not take you more than 30 minutes.

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Let  $a, b, c, d, e$  be distinct integers (but you don't know what they are). Your only access to them is through a magic button. Every time you press the button, you get one of  $a, b, c, d, e$  uniformly at random (with replacement).

For example, the first time you press the button, it might say "7." The next time it might say "9." Then it might say "7" again. Then the next three times it might say "3", "3", "52." At this point, you know that four of the numbers are 7, 9, 3, 52, and you still don't know what the last one is.

Your goal is to learn the minimum of  $a, b, c, d, e$ .

1. What is the expected number of times you push the button before you see the minimum value?
2. What is the probability that you push the button 5 times and don't see the minimum?
3. What is the probability that you push the button 10 times and don't see the minimum?
4. Design a randomized algorithm that uses  $O(1)$  button-pushes and correctly returns  $\min\{a, b, c, d, e\}$  with probability at least  $2/3$ .