

$$T(n) = 4T\left(\frac{n}{2}\right) + O(n)$$

$$T(n) = O(n^2)$$

$$T(n) = 3T\left(\frac{n}{2}\right) + O(n)$$

$$T(n) = O(n^{\log_2 3})$$

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n)$$

$$T(n) = O(n \log n)$$

$$T(n) = T\left(\frac{n}{2}\right) + O(n)$$

$$T(n) = O(n)$$

$$c \cdot n^d + c \cdot a \left(\frac{n}{b}\right)^d + c \cdot a^2 \left(\frac{n}{b^2}\right)^d + \dots + a^{\log_b n} \cdot c$$

$$= c \sum_{t=0}^{\log_b n} a^t \left(\frac{n}{b^t}\right)^d$$

$$= c \cdot n^d \sum_{t=0}^{\log_b n} \left(\frac{a}{b^d}\right)^t$$

$$\begin{aligned} n^d \left(\frac{a}{b^d}\right)^{\log_b n} &= n^d \left(b^{\log_b (a/b^d)}\right)^{\log_b n} \\ &= n^d b^{\log_b (a/b^d) \cdot \log_b n} \\ &= n^d \left(b^{\log_b n}\right)^{\log_b (a/b^d)} \\ &= n^d \cdot n^{\log_b (a/b^d)} \\ &= n^d n^{\log_b a - d} \\ &= n^{\log_b a} \end{aligned}$$