

Pre-lecture exercises will not be collected for credit. However, you will get more out of each lecture if you do them, and they will be referenced during lecture. We recommend **writing out** your answers to pre-lecture exercises before class. Pre-lecture exercises usually should not take you more than 30 minutes.

In this pre-lecture exercise, we will remember a little bit of probability!

1. Let X be a random variable which is 1 with probability $1/100$ and 0 with probability $99/100$.
 - (a) What is the expected value $\mathbb{E}[X]$?
 - (b) Suppose you draw n independent random variables, X_1, X_2, \dots, X_n , distributed like X . What is the expected value $\mathbb{E}[\sum_{i=1}^n X_i]$?
 - (c) Suppose we draw independent random variables X_1, X_2, \dots and we stop when we see the first 1. For example, if we draw

$$X_1 = 0, X_2 = 0, X_3 = 0, X_4 = 1, \dots$$

then we would stop at X_4 . Let N be the last index that we draw; so in the previous example, $N = 4$. How big do you expect N to be?

Note: Actually figuring out $\mathbb{E}[N]$ from scratch is a bit tricky, although you may have seen it in CS 109. But even if you don't do it rigorously, intuitively how big do you expect N to be?

2. Consider the following pseudocode, which sorts an array A . Someone else has implemented the random shuffling for us, so do not worry about the details of that procedure.

```
BogoSort(A) :  
while  $A$  is not sorted do  
  | randomly shuffle  $A$   
return  $A$ 
```

- (a) Let X_i be a random variable which is 1 if A is sorted after the i -th call, and 0 otherwise.
- (b) What is $\mathbb{E}[X_i]$?
- (c) What is the *expected* number of times that BogoSort executes the while loop?