$b(x)$ = #black nodes on path from x to NIL (excl. x, incl. NIL)

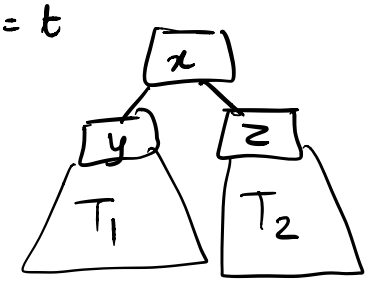
Claim: $\forall x$ # non-NIL nodes under x (include x) $\geq 2^{b(x)} - 1$

I.H: Claim holds for trees of ht $\leq K$

Base Case: $K = 0$ Check yourself!

Inductive Step: Say IH holds for $K = t-1$

Want to show holds for $K = t$



Observe: $b(y)$ $b(x)$ or $b(x)-1$

$b(y) \geq b(x) - 1$

$b(z) \geq b(x) - 1$

$$\#\text{nodes in whole tree} = 1 + \left(\begin{smallmatrix}\#\text{nodes} \\ \text{in } T_1\end{smallmatrix}\right) + \left(\begin{smallmatrix}\#\text{nodes} \\ \text{in } T_2\end{smallmatrix}\right)$$

$$\geq 1 + (2^{b(y)} - 1) + (2^{b(z)} - 1)$$

$$\geq 1 + (2^{b(x)-1} - 1) + (2^{b(x)-1} - 1)$$

$$\geq 2 \cdot 2^{b(x)-1} - 1$$

$$\geq 2^{b(x)} - 1$$

$2^{-1} = \frac{1}{2}$