1 Definitions

Which of the following is the correct English description of \( f(n) = O(g(n)) \)?

- For every constant \( c > 0 \), there is an \( n_0 \), such that for all \( n \geq n_0 \) we have \( f(n) \leq c \cdot g(n) \).
- There is some \( c > 0 \) and some \( n_0 \), such that for all \( n \geq n_0 \) we have \( f(n) \leq c \cdot g(n) \).
- For every \( n_0 \), there is some constant \( c > 0 \) such that for all \( n \geq n_0 \) we have \( f(n) \leq c \cdot g(n) \).

Correct:

Suppose that \( g(n) > 0 \) for all integers \( n \). Then is \( f(n) = O(g(n)) \) equivalent to the following simpler definition that avoids \( n_0 \)?

\[ \exists c > 0 : \forall n \ f(n) \leq c \cdot g(n) \]

- Yes
- No

Correct:

Suppose that \( f(n) = O(g(n)) \). Which of the following is implied by this fact?

- \( g(n) = \Omega(f(n)) \)
- \( g(n) = O(f(n)) \)
- Both
- Neither

Correct:

If \( f(n) = O(g(n)) \), is it true that \( 2^f(n) = O(2^g(n)) \)?

- Yes
- No

Correct:

2 Examples

What is the smallest exponent \( x \) such that \( n^2 + n^3 - n = O(n^x) \)?

- 3

Correct:

Which of the following describes \( n(n + 1)(n + 2)/6 \)?

- \( O(n^4) \)
- \( O(n^3) \)
- \( \Theta(n^2) \)
- \( \Omega(n^2) \)
- All of the above

Correct:

For which exponents \( x \) is \( n(n + 1)/2 = \Theta(n^x) \)?

- 1
- 2
- 3
- All of the above

Correct:

For which function \( g(n) \) is it true that \( n^2 = O(g(n)) \)?

- \( g(n) = 1.01^n \)
- \( g(n) = 2^n \cdot \sin(\pi n/2) \)
- \( g(n) = 2^n \cdot \cos(\pi n/2) \)
- All of the above

Correct: