Selection

In the select algorithm, the runtime is represented with the recurrence relation

\[ T(n) = O(n) + T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right). \]

Here, \( T\left(\frac{n}{5}\right) \) is for selecting the pivot, and \( T\left(\frac{7n}{10}\right) \) is for the recursive call to select the \( k \)-th element.

Consider the modified version of the select algorithm, where we split our array into \( \lceil \frac{n}{7} \rceil \) groups of size \( \leq 7 \) instead. What would be the recurrence relation for this modified version? Specifically, if we write the recurrence relation as

\[ T(n) = O(n) + T\left(\frac{n}{a}\right) + T\left(\frac{bn}{c}\right), \]

where \( a, b, \) and \( c \) are non-negative integers, what are the smallest possible values of \( a, b, \) and \( c \)?

\[ a = \boxed{7} \quad \text{Correct} \]

\[ b = \boxed{5} \quad \text{Correct} \]

\[ c = \boxed{7} \quad \text{Correct} \]

What is the smallest exponent \( x \) such that the modified version of the select described above on an array of size \( n \) always takes time \( O(n^x) \)?

\[ x = \boxed{1} \quad \text{Correct} \]

Now assume that the \( O(n) \) work per recursive step takes exactly \( n \) units of time on our machine. In other words, suppose that the recurrence relation for the runtime is

\[ T(n) = n + T\left(\frac{n}{a}\right) + T\left(\frac{bn}{c}\right). \]

What is the smallest coefficient \( C \) such that we can use the substitution method to prove that the recurrence relation for the modified select algorithm is \( T(n) \leq Cn \)?

\[ C = \boxed{7} \quad \text{Correct} \]

Now consider another modified version of the select algorithm, where we split our array into \( \lceil n/3 \rceil \) groups of size \( \leq 3 \) instead. What would be the recurrence relation for this modified version? Specifically, if we write the recurrence relation as

\[ T(n) = n + T\left(\frac{n}{3}\right) + T\left(\frac{bn}{c}\right), \]

where \( a, b, \) and \( c \) are non-negative integers, what are the smallest possible values of \( a, b, \) and \( c \)?

\[ a = \boxed{3} \quad \text{Correct} \]

\[ b = \boxed{2} \quad \text{Correct} \]

\[ c = \boxed{3} \quad \text{Correct} \]

Which one is true for the modified select recurrence relation that you came up with in the last part?

- \( T(n) = \Theta(n) \)
- \( T(n) = \Theta(n \log n) \)
- \( T(n) = \Theta(n^2) \)

\[ \text{Correct} \]