# Selection

Reset Progress Reveal Solutions

In the select algorithm, the runtime is represented with the recurrence relation

$$T(n) = O(n) + T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right).$$

Here, $T(\frac{n}{5})$ is for selecting the pivot, and $T(\frac{7n}{10})$ is for the recursive call to select the $k$-th element.

Consider the modified version of the select algorithm, where we split our array into $\lceil \frac{n}{7} \rceil$ groups of size $\leq 7$ instead. What would be the recurrence relation for this modified version? Specifically, if we write the recurrence relation as $T(n) = O(n) + T(\frac{n}{a}) + T(\frac{bn}{c})$, where $a$, $b$, and $c$ are non-negative integers, what are the smallest possible values of $a$, $b$, and $c$?

$a =$

7

Correct

$b =$

5

Correct

$c =$

7

Correct

What is the smallest exponent $x$ such that the modified version of the select described above on an array of size $n$ always takes time $O(n^x)$?

1

Correct

Now assume that the $O(n)$ work per recursive step takes exactly $n$ units of time on our machine. In other words, suppose that the recurrence relation for the runtime is

$$T(n) = n + T\left(\frac{n}{a}\right) + T\left(\frac{bn}{c}\right).$$

What is the smallest coefficient $C$ such that we can use the substitution method to prove that the recurrence relation for the modified select algorithm is $T(n) \leq Cn$

7

Correct

Now consider another modified version of the select algorithm, where we split our array into $\lceil n/3 \rceil$ groups of size $\leq 3$ instead. What would be the recurrence relation for this modified version? Specifically, if we write the recurrence relation as

$$T(n) = n + T\left(\frac{n}{a}\right) + T\left(\frac{bn}{c}\right),$$

where $a$, $b$, and $c$ are non-negative integers, what are the smallest possible values of $a$, $b$, and $c$?

$a =$

3

Correct

$b =$

2

Correct

$c =$

3

Correct

Which one is true for the modified select recurrence relation that you came up with in the last part?

- ○ $T(n) = \Theta(n)$
- ● $T(n) = \Theta(n \log n)$
- ○ $T(n) = \Theta(n^2)$

Correct