CS 161 W22: Section 9 Problems

March 2022

Exercise 0

Given a weighted, undirected graph $G = (V, E)$, with edge weights in the set $W = \{1, 2, 3\}$, design an algorithm to find the minimum weight spanning tree with a running time of $O(|E|)$.

Exercise 1

How would you modify the Max-flow algorithm to handle the following situation?

(a) Suppose that instead of a single source and sink s, t respectively, we have multiple sources $S = \{s_1, s_2, \cdots, s_k\}$ and multiple sinks $T = \{t_1, t_2, \cdots, t_l\}$. How can you find the max flow in the graph from sources to sinks?

Exercise 2

You've gone on a trip with k friends, where friend i paid c_i for the group's expenses. You would like to develop an algorithm to ensure that everyone gets paid back fairly, but without going through one person. That is, each person should either pay or receive money, but not both. Devise an algorithm to determine how much money each person should pay or receive.

Exercise 3

Given a set of n cities, we would like to build a transportation system such that there is some path from any city i to any other city j. There are two ways to travel: by driving or by flying. Initially, all of the cities are disconnected. It costs r_{ij} to build a road between city i and city j. It costs a_i to build an airport in city i. For any two cities i and j, we can fly directly from i to j if there is an airport in both cities. Devise an efficient algorithm for determining which roads and airports to build to minimize the cost of connecting the cities.