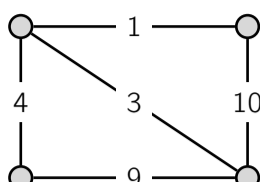


## 1 Conditions for Shortest Path Algorithms

Suppose that we want to find the shortest path between two nodes in the following graph. Which algorithm can we use?



- BFS
- Dijkstra
- Bellman-Ford
- All of the above
- BFS and Dijkstra
- Dijkstra and Bellman-Ford

Correct

We have a graph with negative edge weights. Can we use Dijkstra to find shortest paths?

- Yes
- No

Correct

We have an undirected graph with positive edge weights. Can we use Dijkstra to find shortest paths?

- Yes
- No

Correct

We have a directed graph with positive edge weights. Can we use Dijkstra to find shortest paths?

- Yes
- No

Correct

## 2 Dijkstra Forensics

Suppose we run Dijkstra on some graph with nodes  $A, B, C, D, E, F$  that has nonnegative ( $\geq 0$ ) edge weights, starting from the node  $A$ . In the middle of the algorithm our computer crashes. We look through the memory dump, and see that the state of  $d$  looked as follows when the crash happened:

$$d[A] = 0, d[B] = 5, d[C] = 4, d[D] = 15, d[E] = 2, d[F] = 20.$$

Additionally from the memory dump we see that the current node when the crash happened was node  $C$ .

What is the minimum possible length of the shortest path from node  $A$  to node  $B$ ?

Correct

What is the maximum possible length of the shortest path from node  $A$  to node  $B$ ?

Correct

What is the minimum possible length of the shortest path from node  $A$  to node  $D$ ?

Correct

What is the maximum possible length of the shortest path from node  $A$  to node  $D$ ?

Correct

What is the minimum possible length of the shortest path from node  $A$  to node  $E$ ?

Correct

What is the maximum possible length of the shortest path from node  $A$  to node  $E$ ?

Correct

What is the minimum possible length of the shortest path from node  $A$  to node  $F$ ?

Correct

What is the maximum possible length of the shortest path from node  $A$  to node  $F$ ?

Correct

If we run the Dijkstra algorithm on the graph of U.S. streets/roads/highways/etc., starting from the Stanford Oval, which of the following locations will become the current node first?

- Times Square in New York
- The Hollywood Sign
- Tresidder Union
- The ordering might differ in each run of Dijkstra.

Correct

## 3 Runtime

Suppose that we implement Dijkstra with a red-black tree. What is the asymptotically smallest upper bound on runtime in terms of  $n$  (the number of nodes) and  $m$  (the number of edges).

- $O(n \log n + m)$
- $O((n + m) \log n)$
- $O(n + m)$

Correct

What if we implement Dijkstra with using a Fibonacci heap? What is the asymptotically smallest upper bound on runtime in terms of  $n$  (the number of nodes) and  $m$  (the number of edges).

- $O(n \log n + m)$
- $O((n + m) \log n)$
- $O(n + m)$

Correct

Suppose that we have a heap data structure that does not support updating the keys (many standard implementations of heaps in various programming languages do not support the update key operation).

Our data structure keeps a collection of items, each of form (key, object), where keys are numbers, and object can be anything (we will store vertices as our objects). Our data structure supports two operations:

- Insert a new (key, object) into the collection.
- Remove the item with the lowest key currently in the collection and return the key and object for it.

We run a modification of Dijkstra with the following pseudo-code:

```

d ← array indexed with vertices and filled with ∞
H ← empty heap
Insert (0, starting node) into H
while H is not empty do
  Remove (key, vertex) from H with the smallest key.
  if key < d[vertex] then
    d[vertex] ← key
    for all neighbors w of vertex do
      Insert (d[vertex] + weight(vertex, w), w) into H.
    
```

What is the asymptotically smallest upper bound on the runtime of the above code assuming that both the insert and remove operations on  $H$  take  $O(\log(\text{size of } H))$  time? Assume that  $n - 1 \leq m \leq n^2$  (in particular  $\log m = \Theta(\log n)$ ).

- $O(n \log n + m)$
- $O(m \log n)$
- $O(n + m)$

Correct