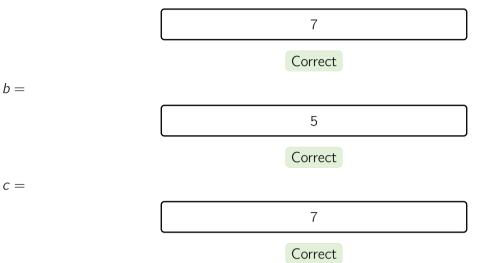
In the select algorithm, the runtime is represented with the recurrence relation

$$T(n) = O(n) + T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right).$$

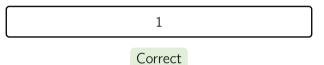
Here, $T(\frac{n}{5})$ is for selecting the pivot, and $T(\frac{7n}{10})$ is for the recursive call to select the k-th element.

Consider the modified version of the select algorithm, where we split our array into $\lceil \frac{n}{7} \rceil$ groups of size ≤ 7 instead. What would be the recurrence relation for this modified version? Specifically, if we write the recurrence relation as $T(n) = O(n) + T(\frac{n}{a}) + T(\frac{bn}{c})$, where *a*, *b*, and *c* are non-negative integers, what are the smallest possible values of *a*, *b*, and *c*?

a =



What is the smallest exponent x such that the modified version of the select described above on an array of size n always takes time $O(n^x)$?



Now assume that the O(n) work per recursive step takes exactly n units of time on our machine. In other words, suppose that the recurrence relation for the runtime is

$$T(n) = n + T\left(\frac{n}{a}\right) + T\left(\frac{bn}{c}\right).$$

What is the smallest coefficient C such that we can use the substitution method to prove that the recurrence relation for the modified select algorithm is $T(n) \leq Cn$



Correct

Now consider another modified version of the select algorithm, where we split our array into $\lceil n/3 \rceil$ groups of size ≤ 3 instead. What would be the recurrence relation for this modified version? Specifically, if we write the recurrence relation as

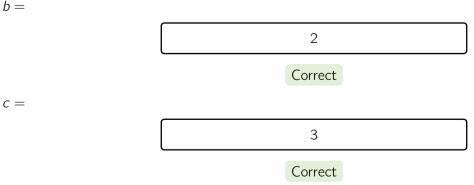
$$T(n) = n + T\left(\frac{n}{a}\right) + T\left(\frac{bn}{c}\right),$$

where a, b, and c are non-negative integers, what are the smallest possible values of a, b, and c?

a =

Correct

3



Which one is true for the modified select recurrence relation that you came up with in the last part? O $T(n) = \Theta(n)$ O $T(n) = \Theta(n \log n)$ O $T(n) = \Theta(n^2)$

Correct