Lecture 9

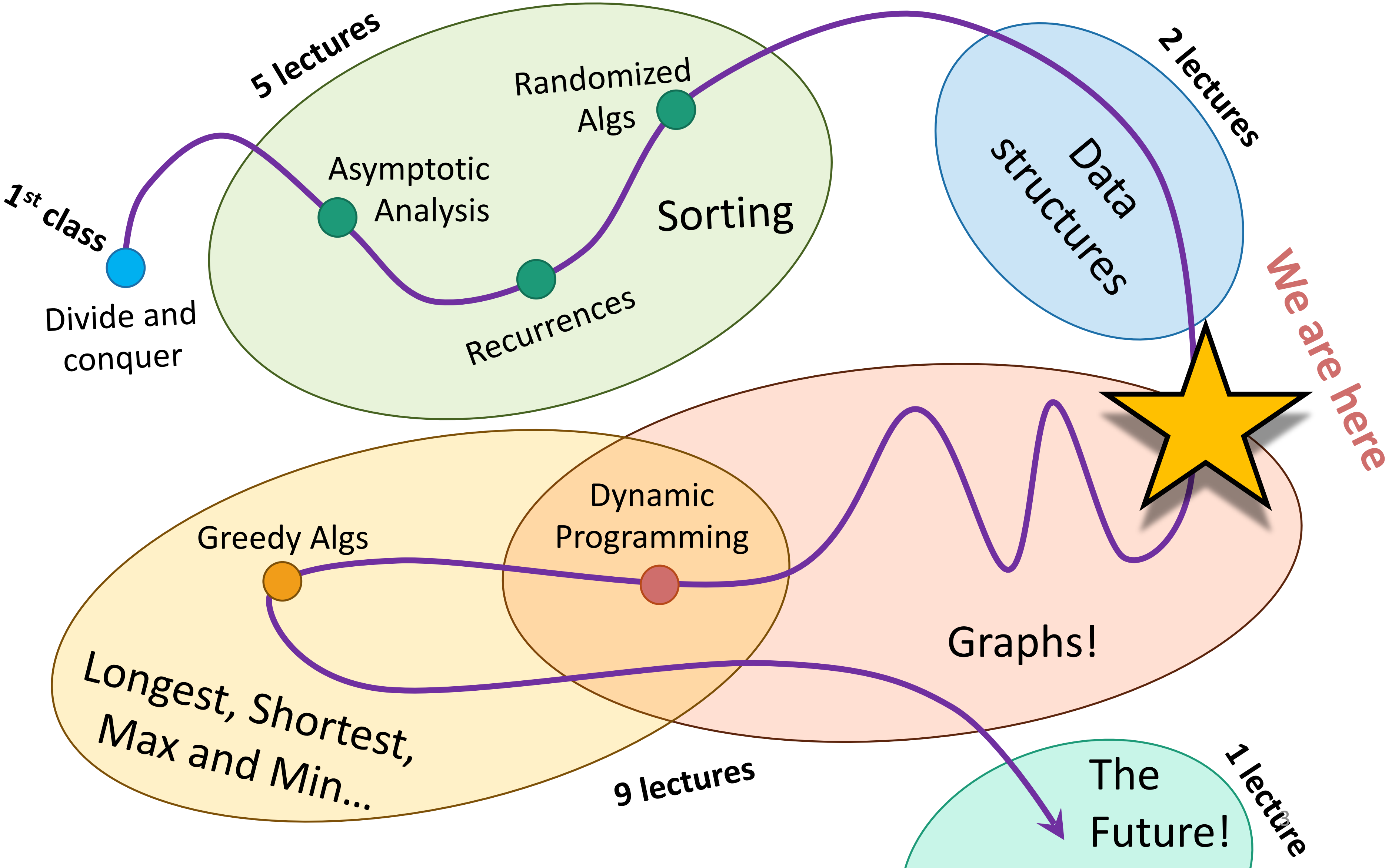
Graphs, BFS and DFS

Announcements!

- Homework 4 due today.
- No new homework this week: use the time to study for the midterm!
- Midterm (Feb 16, 6pm – 9pm) covers up to (and incl.) lecture 7. This week’s lectures are not included.

More detailed schedule on the website!

Roadmap



1st class

Divide and conquer

5 lectures

Asymptotic Analysis

Randomized Algs

Recurrences

Sorting

2 lectures

Data structures

We are here

Dynamic Programming

Greedy Algs

Graphs!

Longest, Shortest, Max and Min…

9 lectures

1 lecture

The Future!

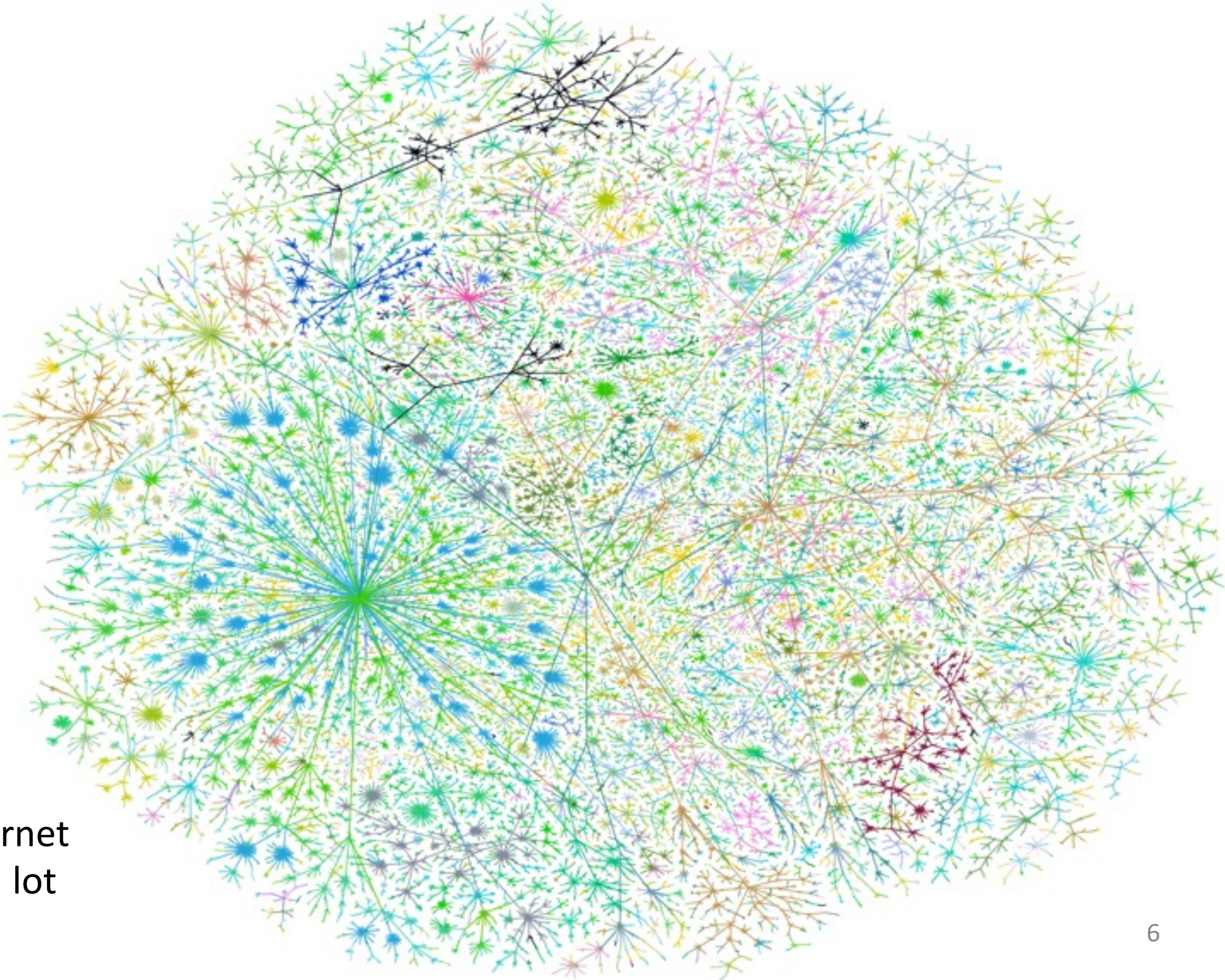
Outline

- Part 0: Graphs and terminology

- Part 1: Depth-first search
    - Application: topological sorting
    - Application: in-order traversal of BSTs
- Part 2: Breadth-first search
    - Application: shortest paths
    - Application (if time): is a graph bipartite?

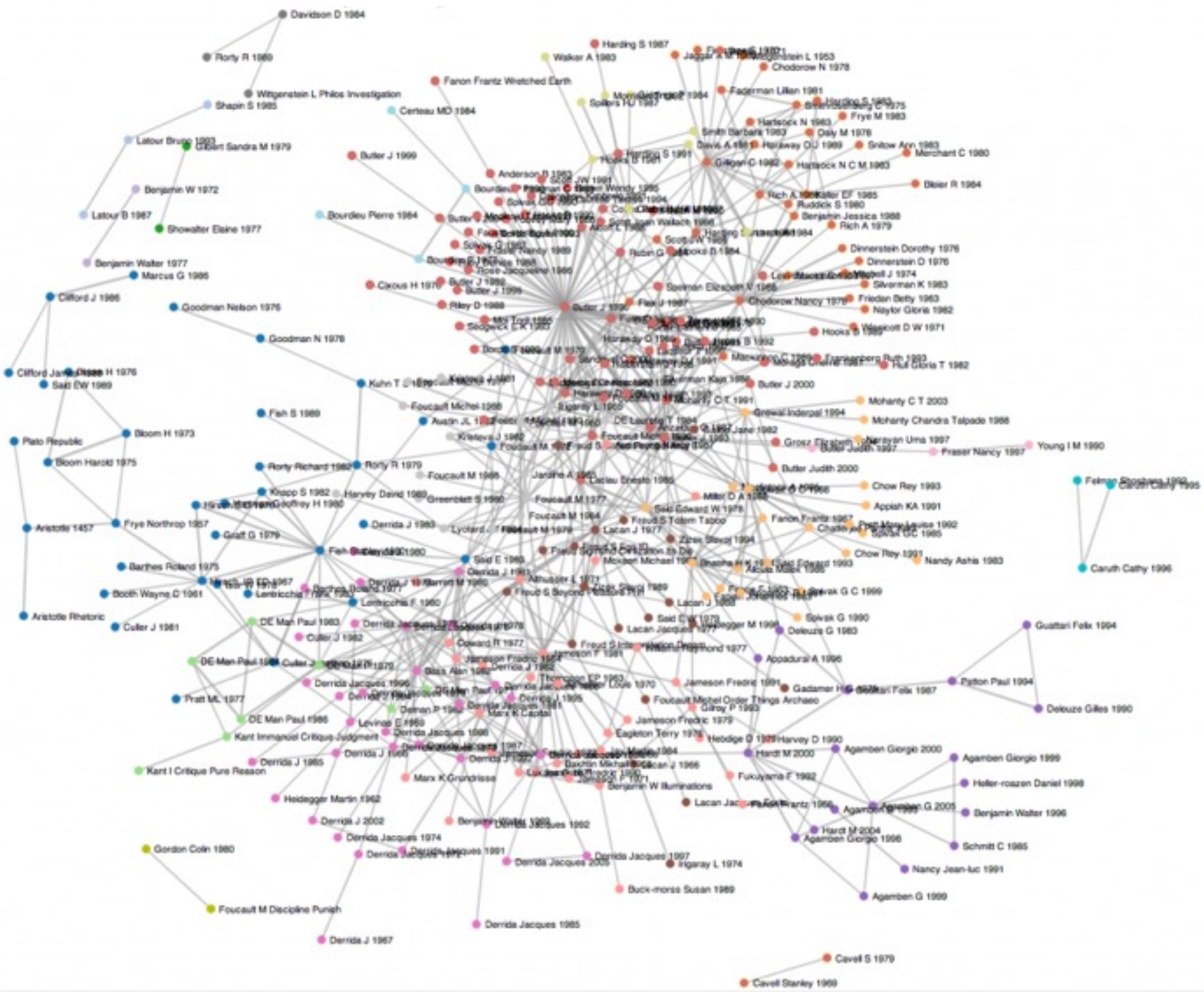
Part 0: Graphs

Graphs



Graph of the internet (circa 1999…it's a lot bigger now…)

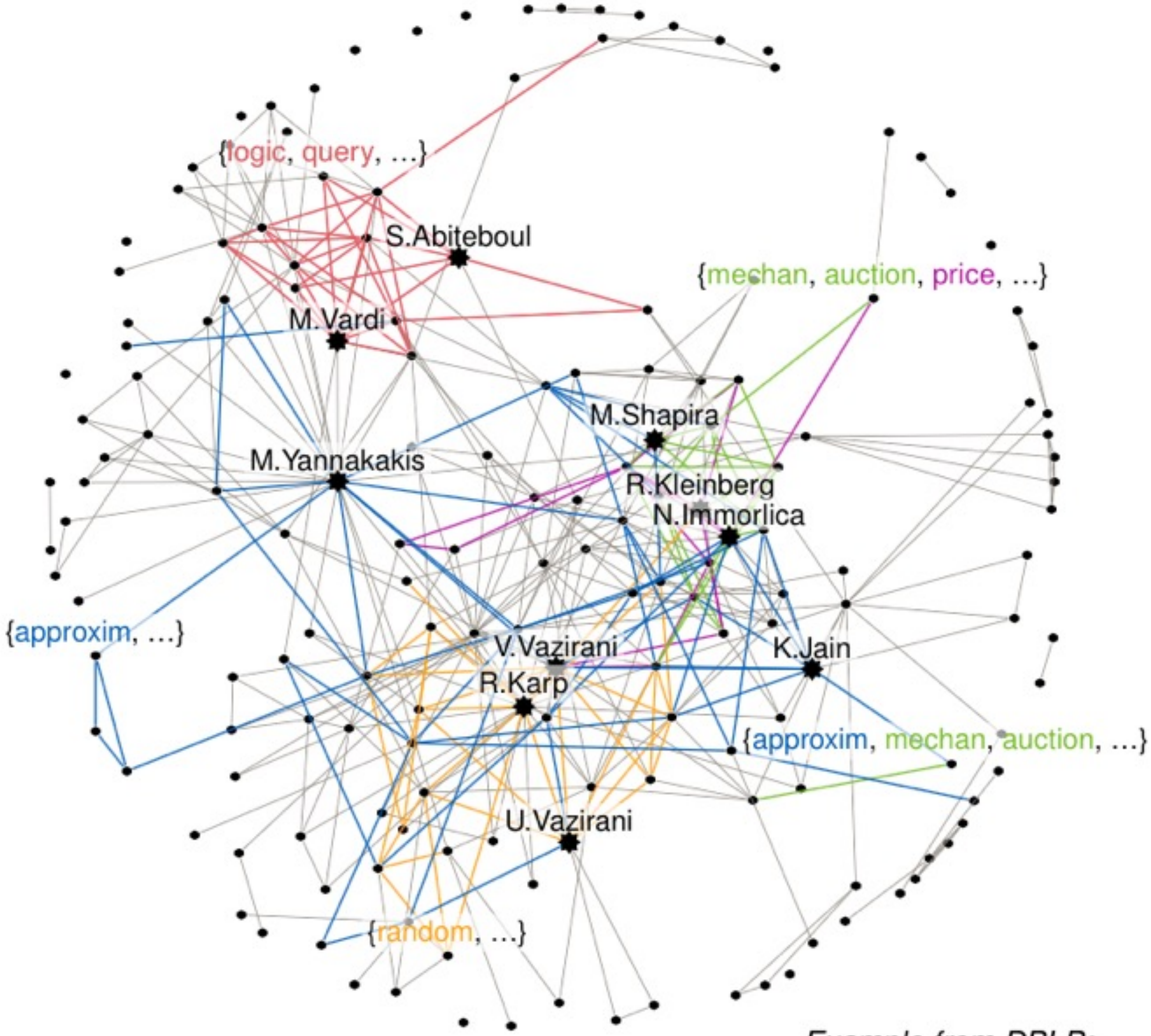
Graphs



Citation graph of literary theory academic papers

Graphs

Theoretical Computer Science academic communities



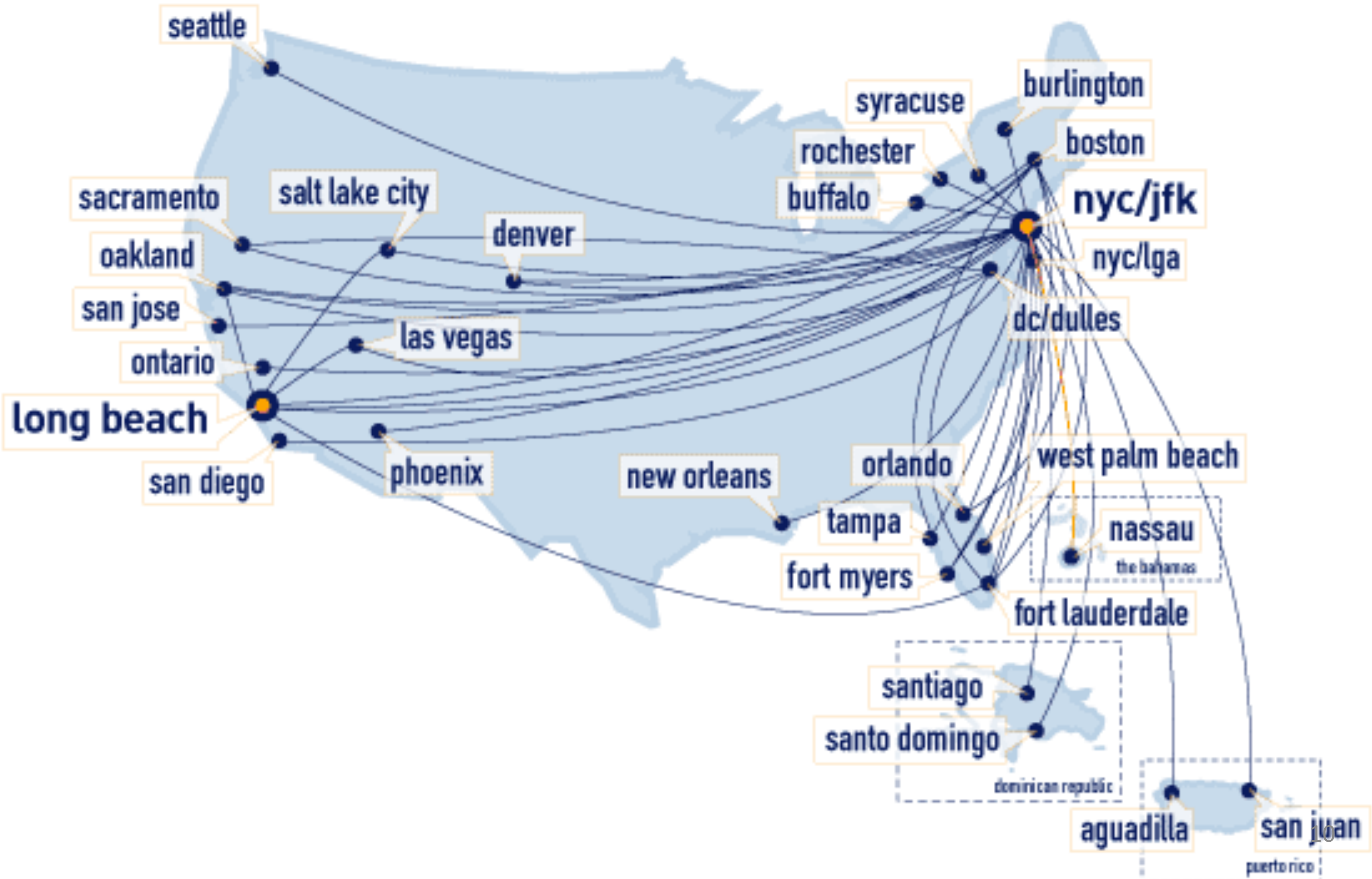
Example from DBLP:
Communities within the co-authors of Christos H. Papadimitriou

Graphs

The Godfather Characters Interaction Network

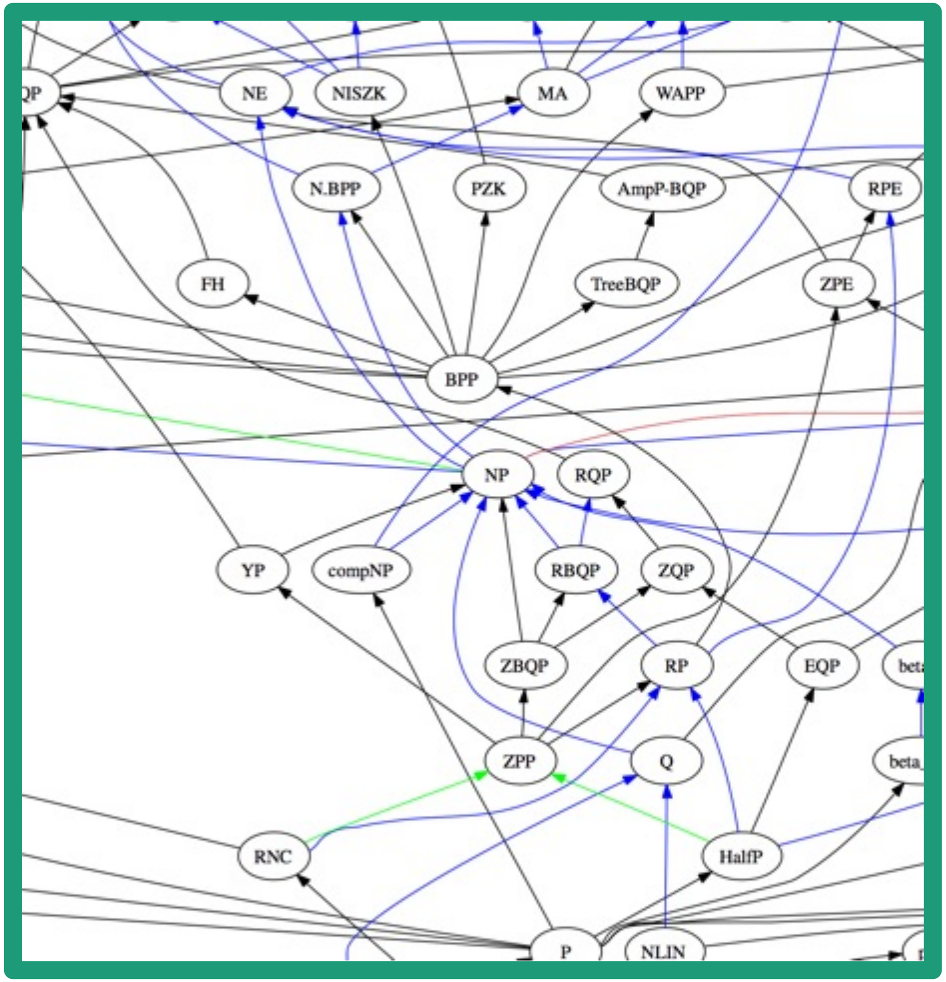
Graphs

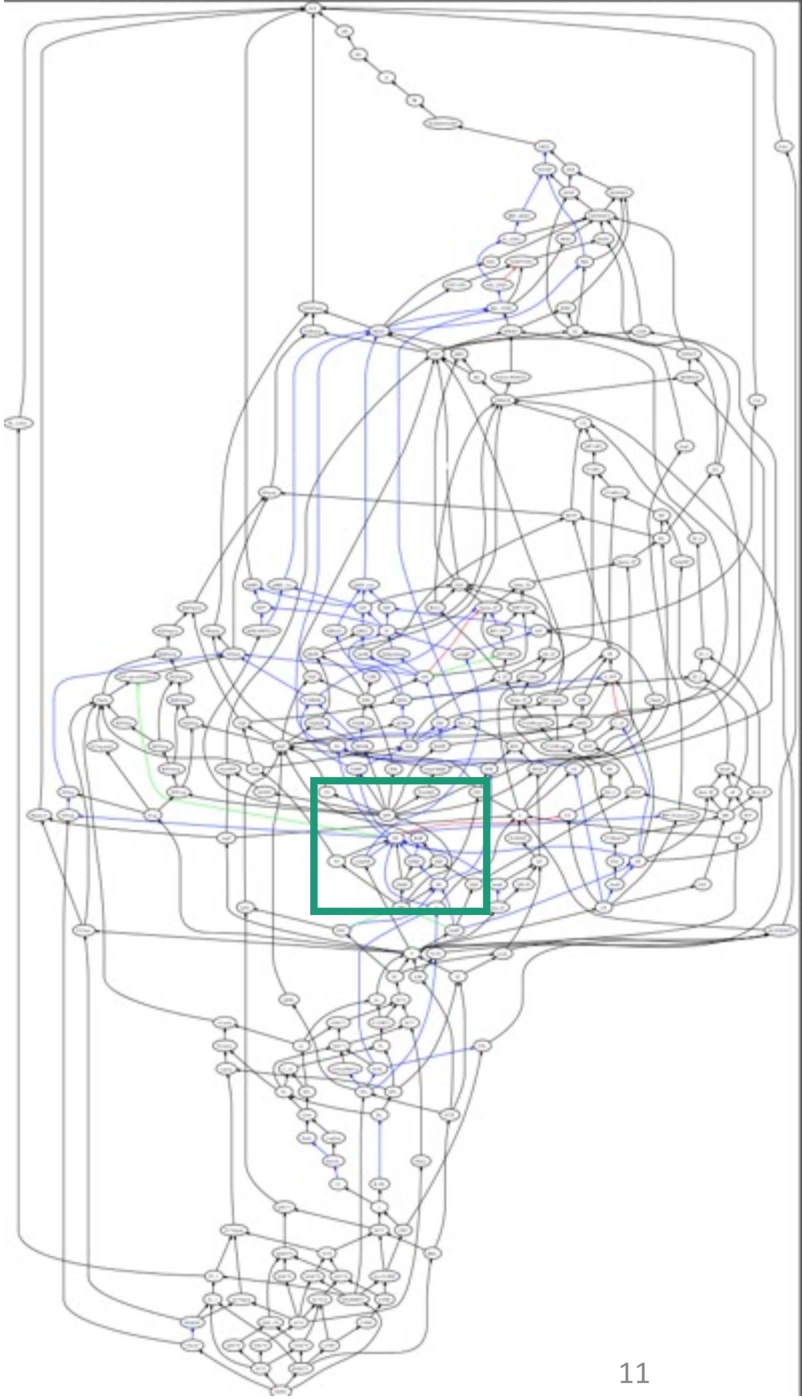
jetblue flights



Graphs

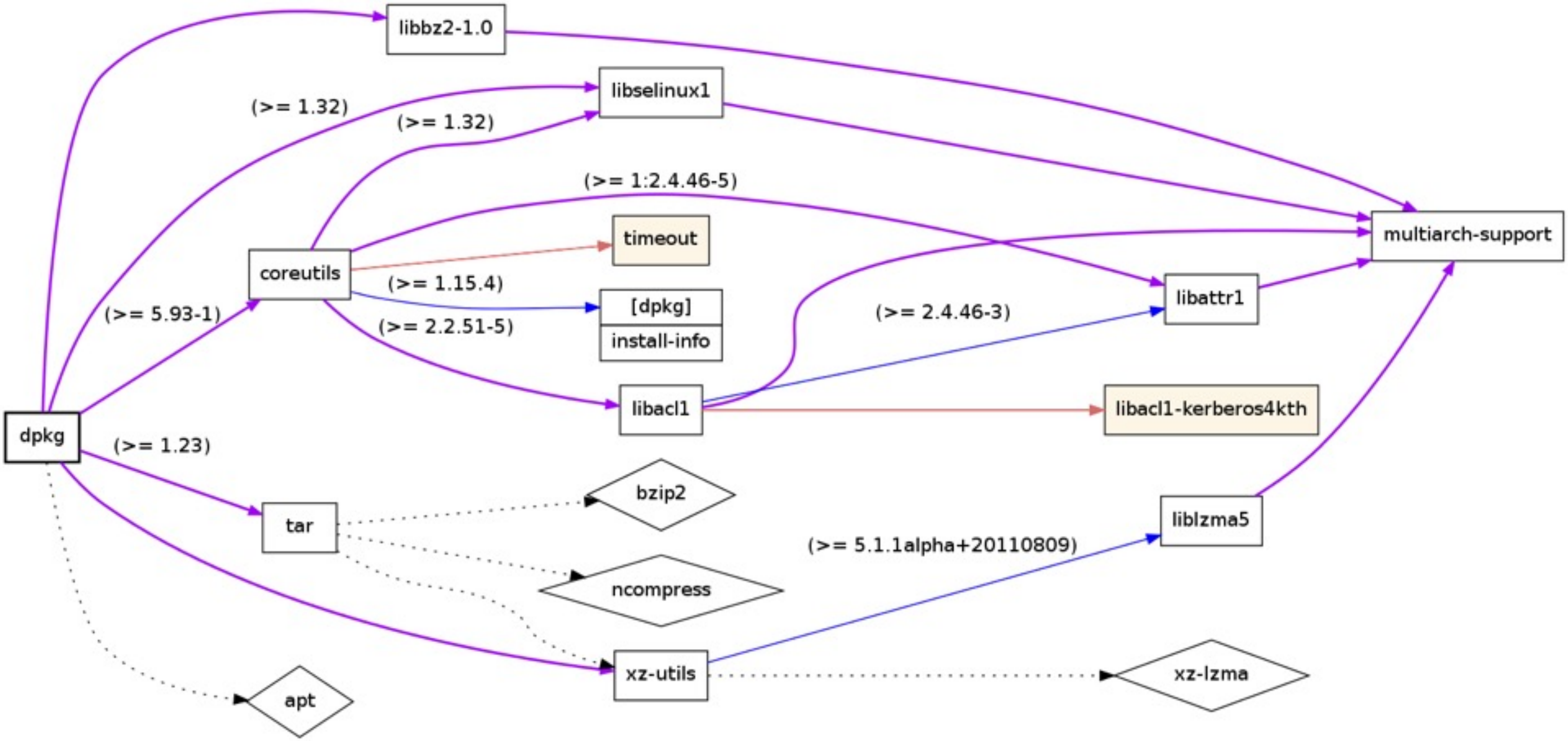
Complexity Zoo containment graph



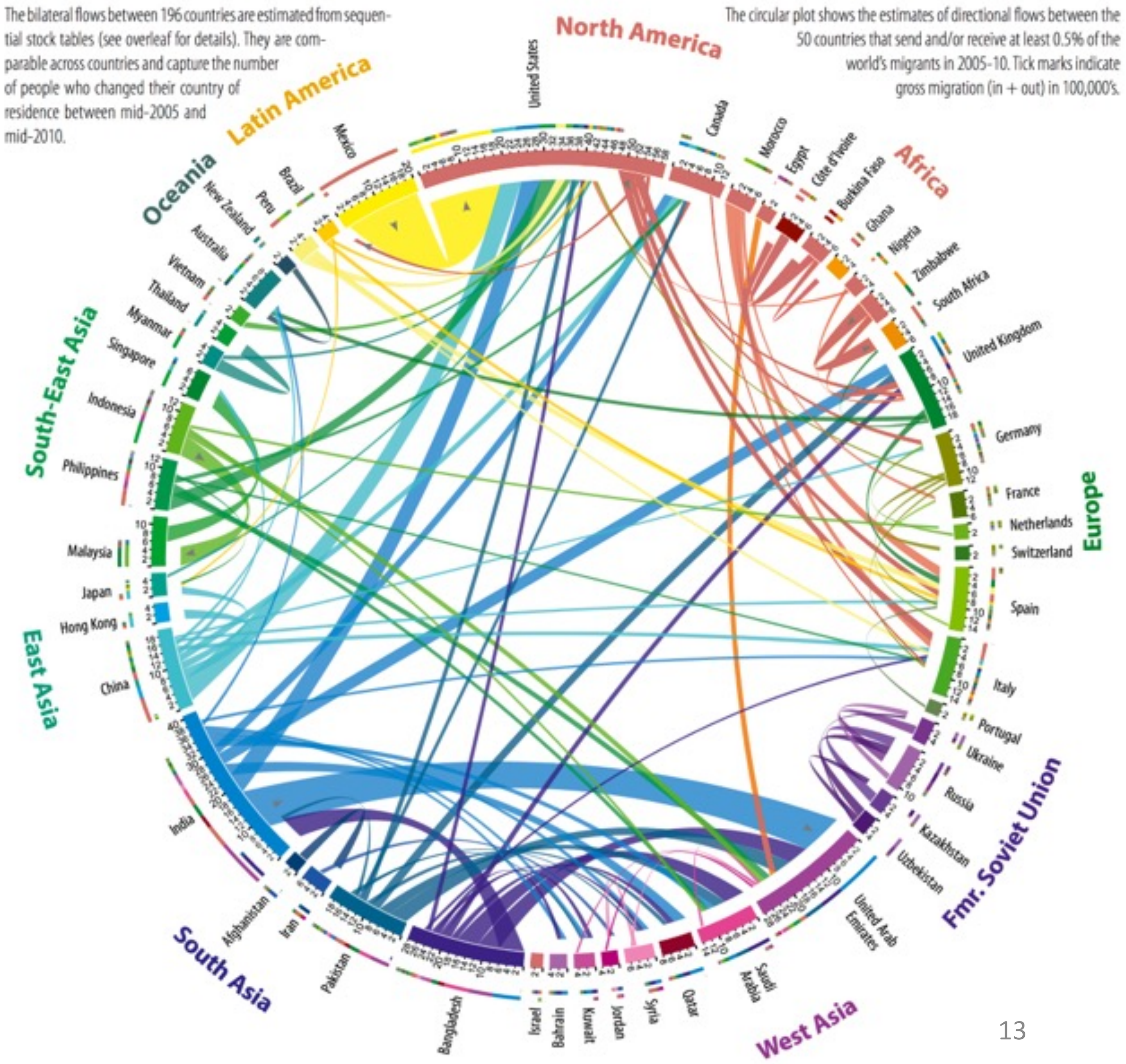


Graphs

debian dependency (sub)graph



Graphs



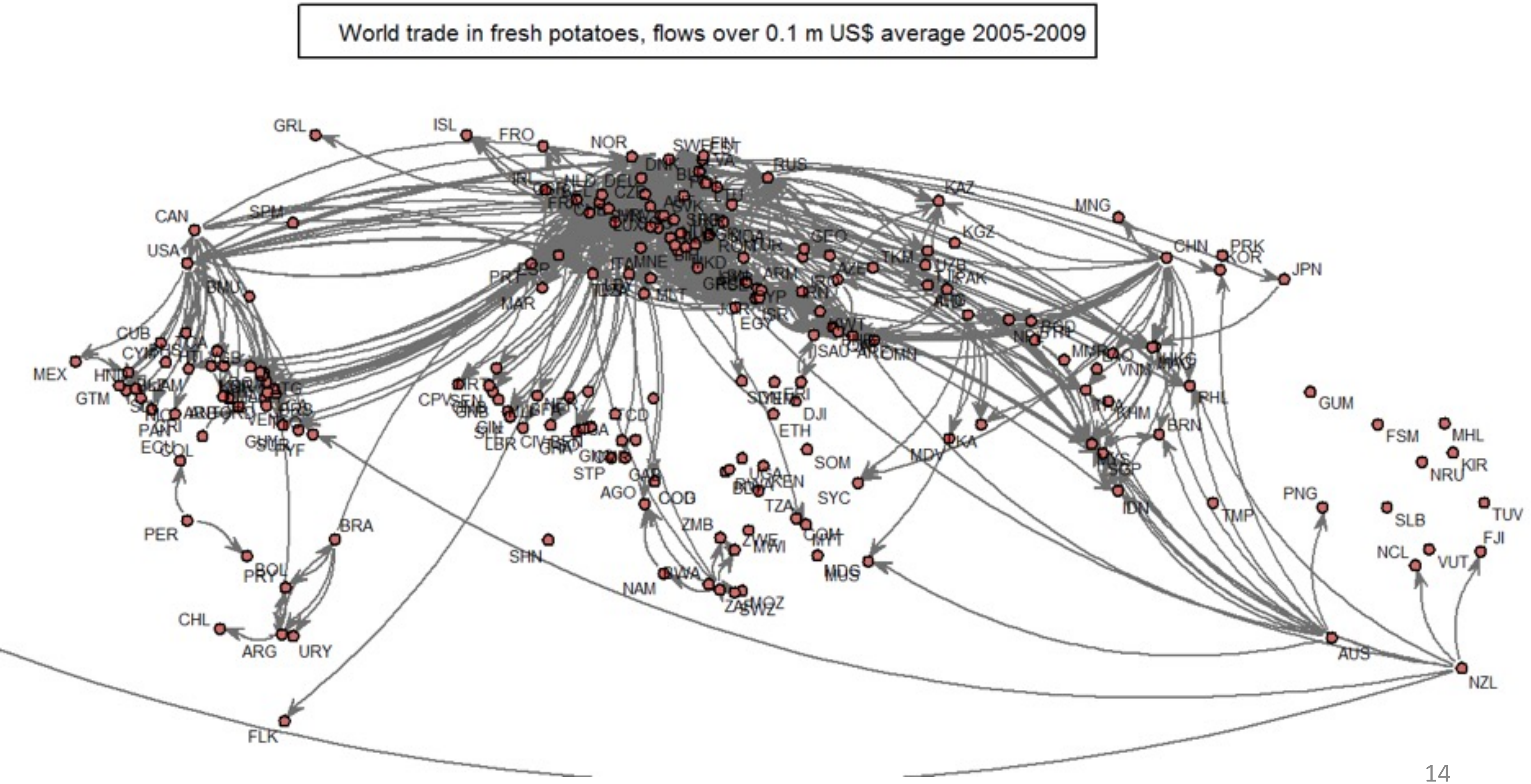
The bilateral flows between 196 countries are estimated from sequential stock tables (see overleaf for details). They are comparable across countries and capture the number of people who changed their country of residence between mid-2005 and mid-2010.

The circular plot shows the estimates of directional flows between the 50 countries that send and/or receive at least 0.5% of the world's migrants in 2005-10. Tick marks indicate gross migration (in + out) in 100,000's.

Immigration flows

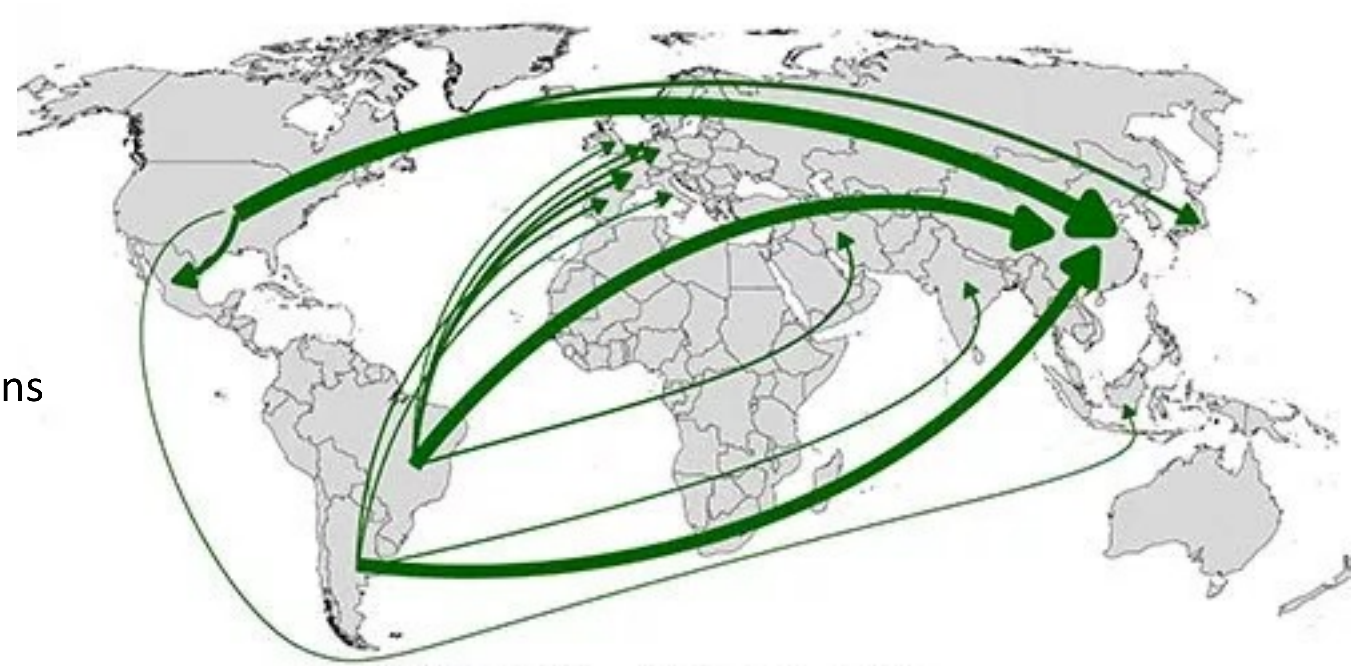
Graphs

Potato trade

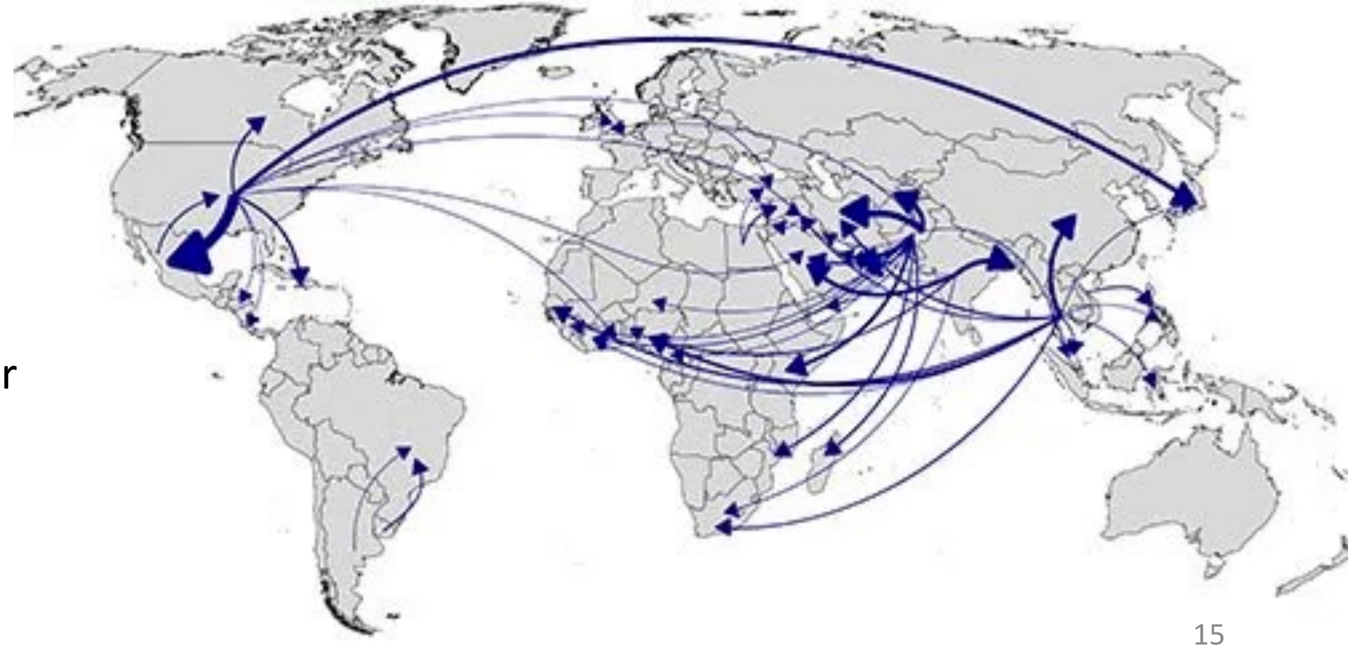


World trade in fresh potatoes, flows over 0.1 m US$ average 2005-2009

Graphs



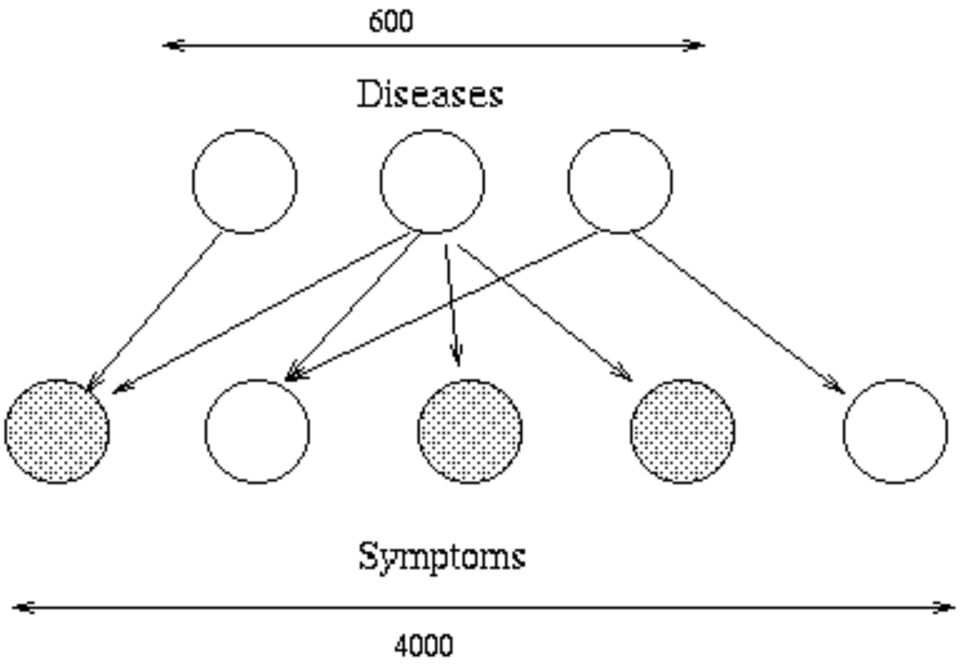
Soybeans



Water

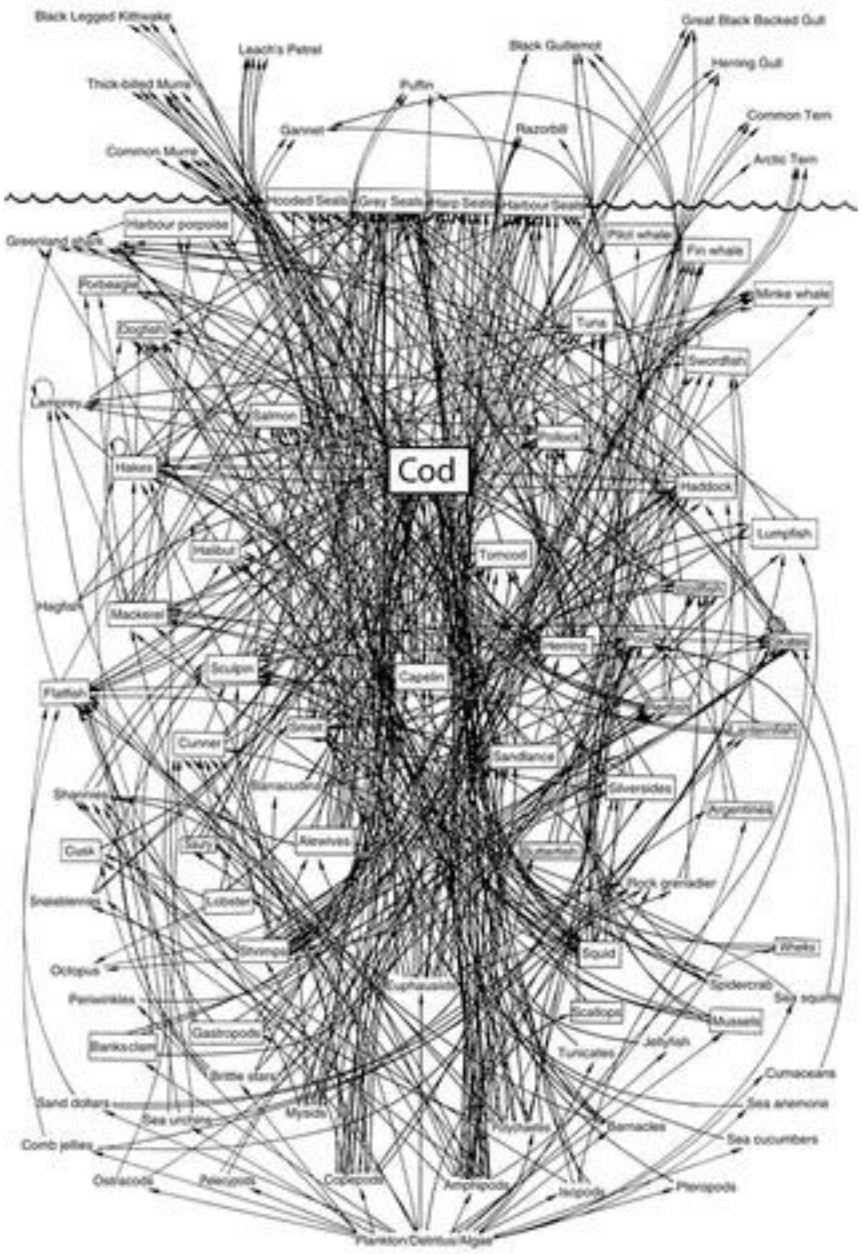
Graphs

Graphical models



Graphs

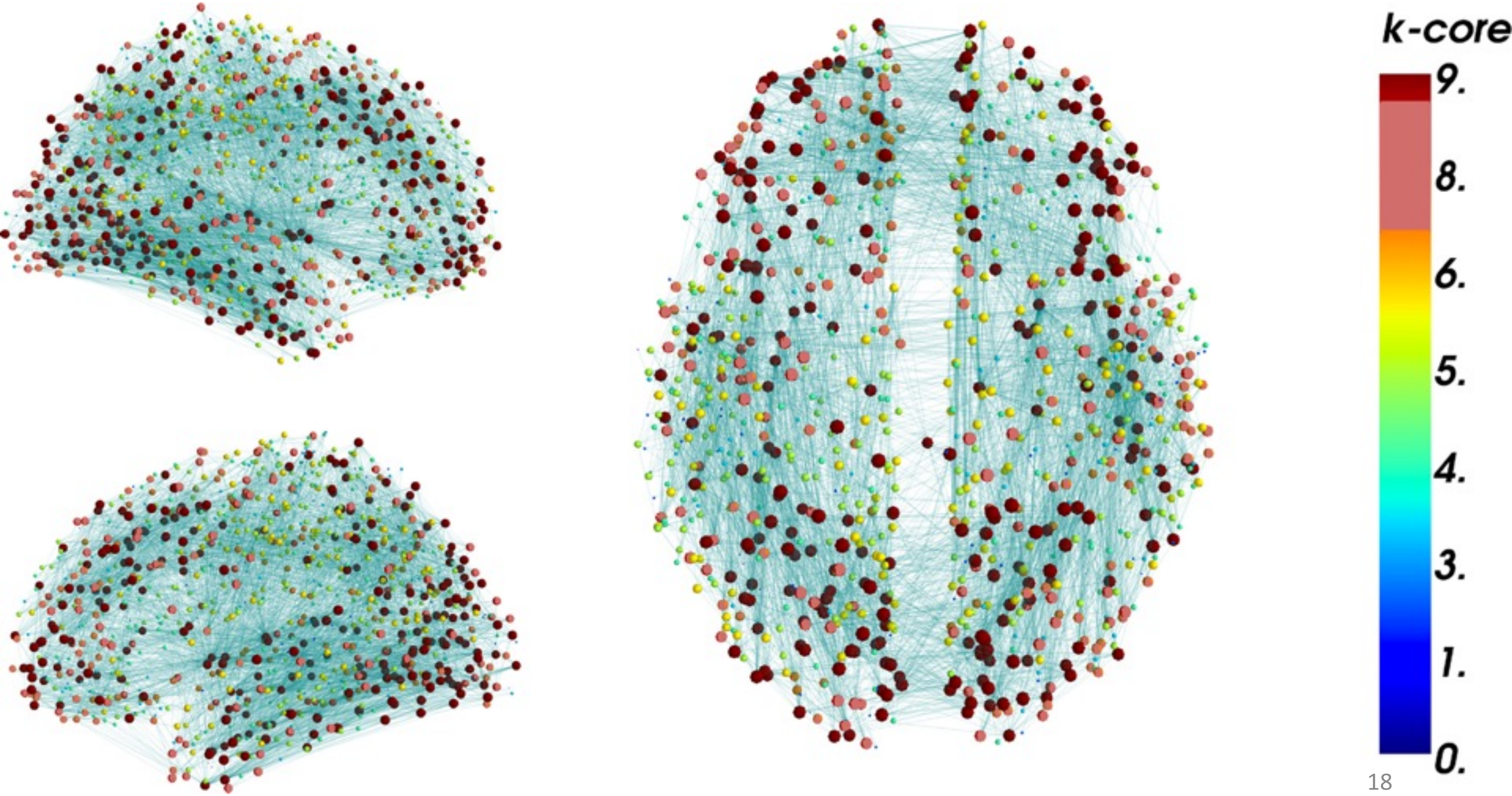
What eats what in the Atlantic ocean?



A simplified food web for the Northwest Atlantic. © IMMA

Graphs

Neural connections in the brain



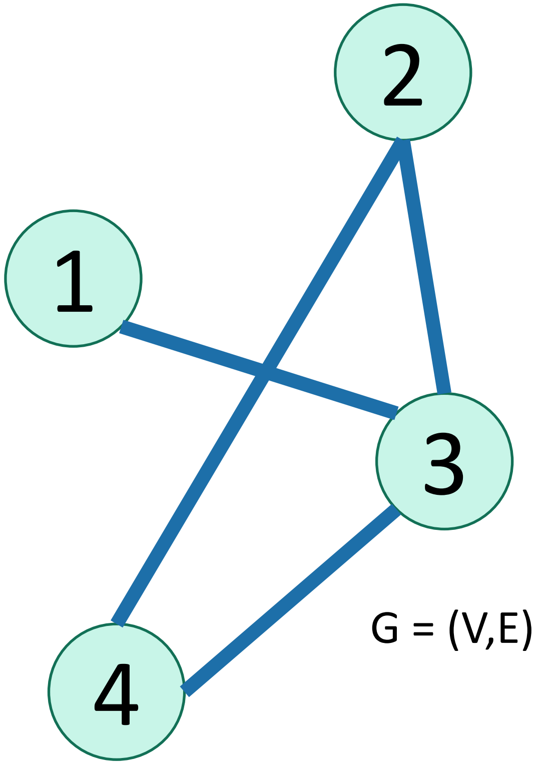
Graphs

- **There are a lot of graphs.**

- We want to answer questions about them.
  - Efficient routing?
  - Community detection/clustering?
  - From pre-lecture exercise:
    - Computing Bacon numbers
    - Signing up for classes without violating pre-req constraints
    - How to distribute fish in tanks so that none of them will fight.

- This is what we'll do for the next several lectures.

Undirected Graphs

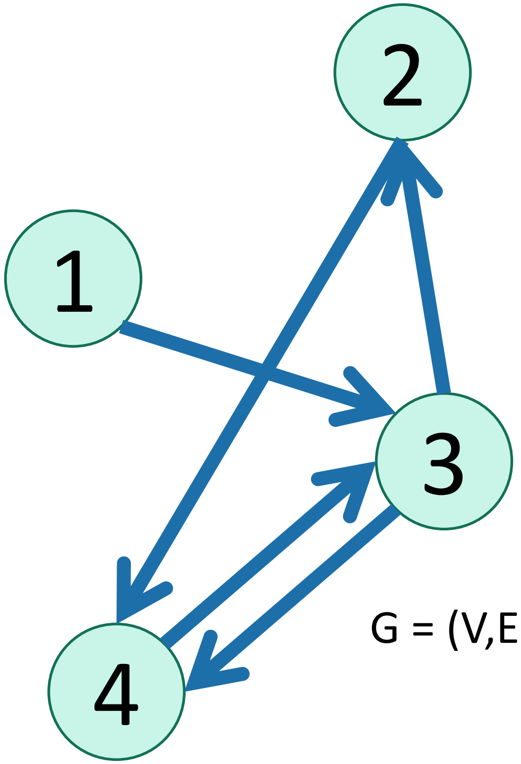


- Has vertices and edges
    - V is the set of vertices
    - E is the set of edges
    - Formally, a graph is G = (V,E)
- Example
    - V = {1,2,3,4}
    - E = { {1,3}, {2,4}, {3,4}, {2,3} }

- The **degree** of vertex 4 is 2.
    - There are 2 edges coming out
- Vertex 4's **neighbors** are 2 and 3

Directed Graphs

- Has vertices and edges
  - V is the set of vertices
  - E is the set of **DIRECTED** edges
  - Formally, a graph is G = (V,E)
- Example
  - V = {1,2,3,4}
  - E = { (1,3), (2,4), (3,4), (4,3), (3,2) }



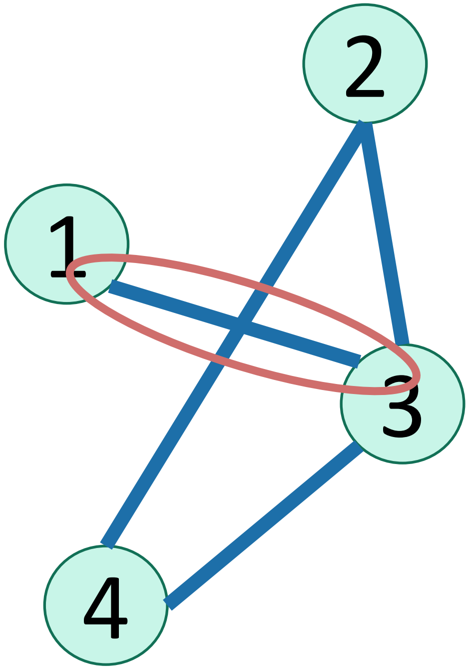
G = (V,E)

- The **in-degree** of vertex 4 is 2.
- The **out-degree** of vertex 4 is 1.
- Vertex 4’s **incoming neighbors** are 2,3
- Vertex 4’s **outgoing neighbor** is 3.

How do we represent graphs?

- Option 1: adjacency matrix

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 & 1 \\ 3 & 1 & 1 & 0 & 1 \\ 4 & 0 & 1 & 1 & 0 \end{array}$$



How do we represent graphs?

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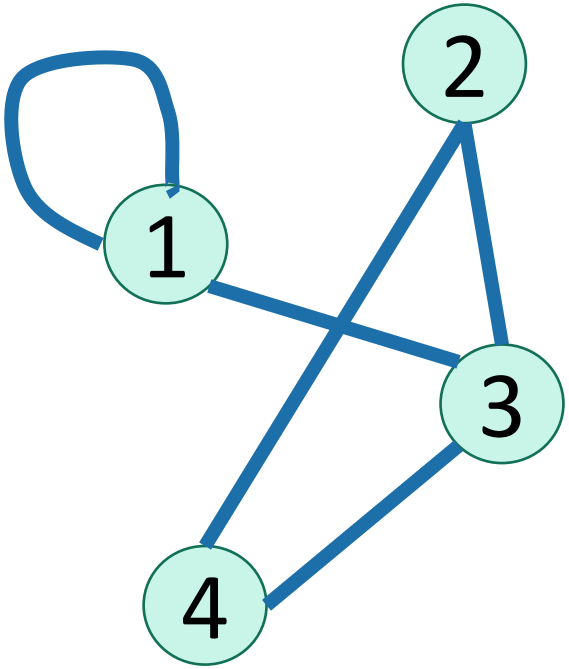


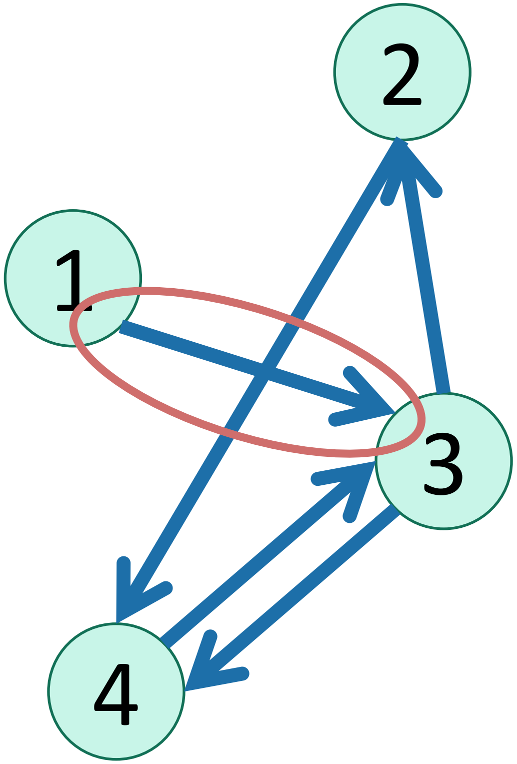
How do we represent graphs?

- Option 1: adjacency matrix

Destination

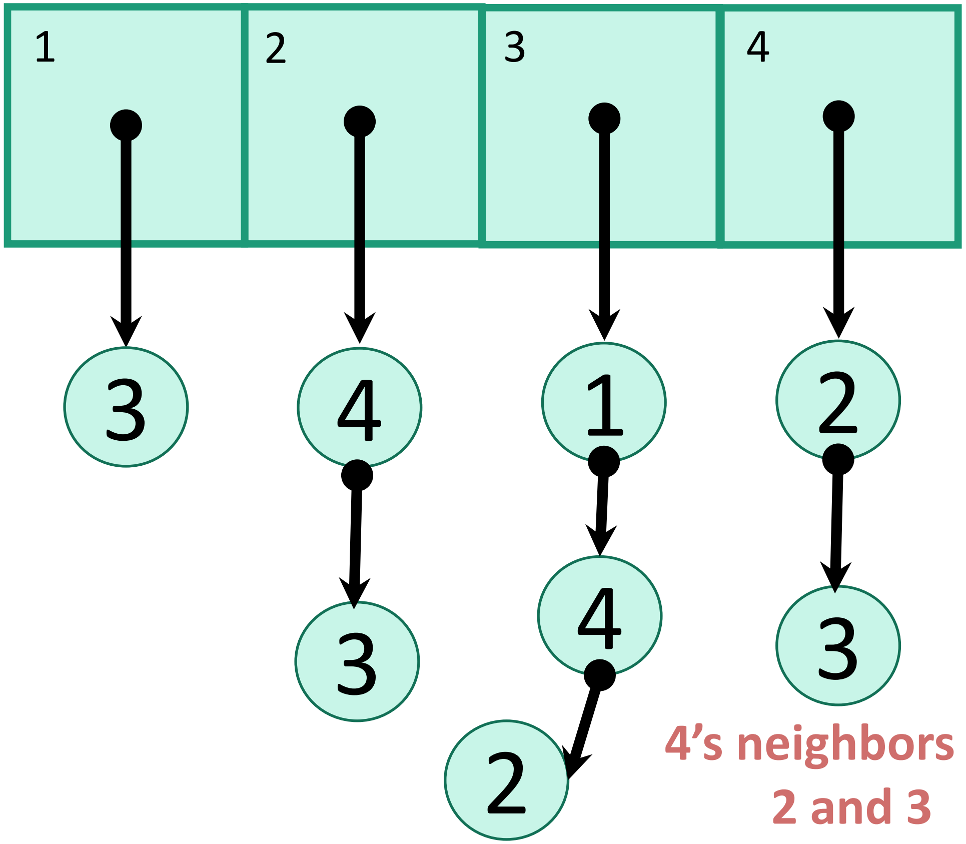
Source

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 0 & 1 & 0 & 1 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

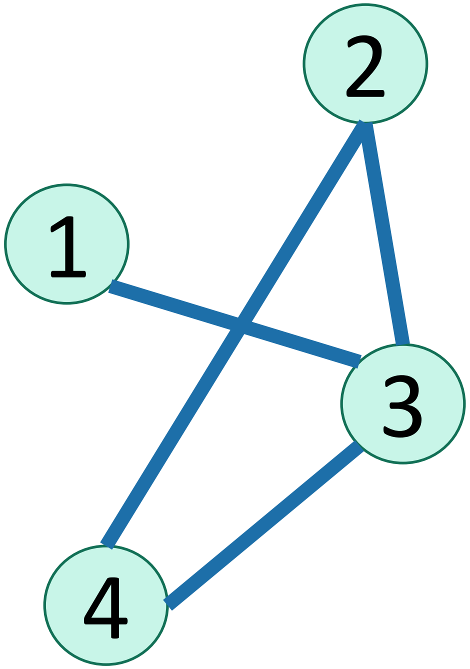


How do we represent graphs?

- Option 2: adjacency lists.



4's neighbors are 2 and 3



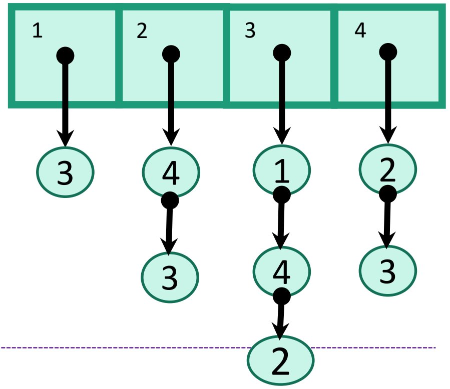
How would you modify this for directed graphs?

In either case

- Vertices can store other information
  - Attributes (name, IP address, …)
  - Helper info for algorithms that we will perform on the graph

- Want to be able to do the following operations:
  - **Edge Membership**: Is edge e in E?
  - **Neighbor Query**: What are the neighbors of vertex v?

Trade-offs

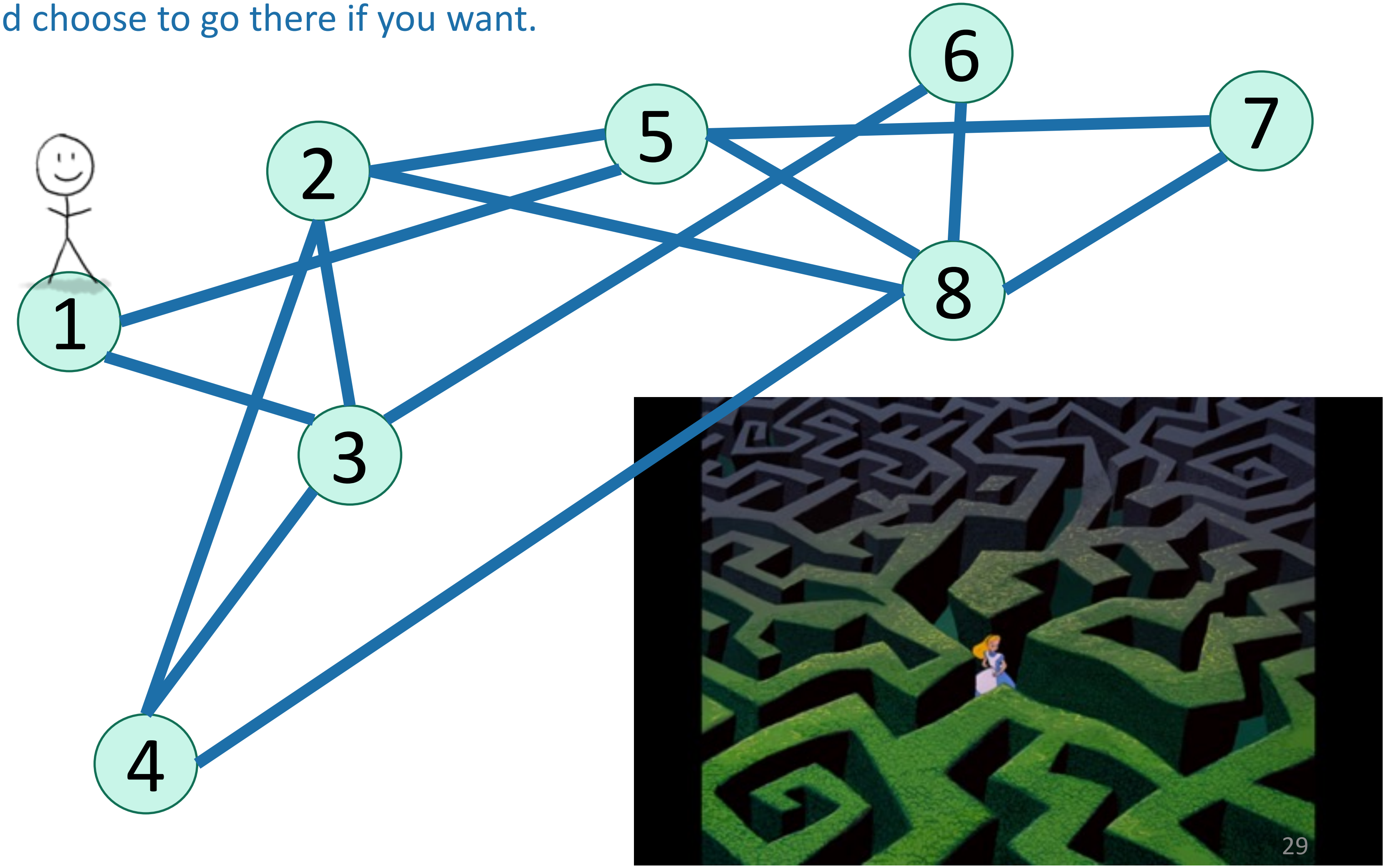
Say there are n vertices and m edges.	$\begin{bmatrix}0 & 0 & 1 & 0\\0 & 0 & 1 & 1\\1 & 1 & 0 & 1\\0 & 1 & 1 & 0\end{bmatrix}$	Generally better for **sparse** graphs (where $m \ll n^2$) 
Edge membership Is e = {v,w} in E?	O(1)	O(deg(v)) or O(deg(w))
Neighbor query Give me a list of v's neighbors.	O(n)	O(deg(v))
Space requirements	O(n^2)	O(n + m) **We'll assume this representation for the rest of the class**

See Lecture 9 Python notebook for an actual implementation!

Part 1: Depth-first search

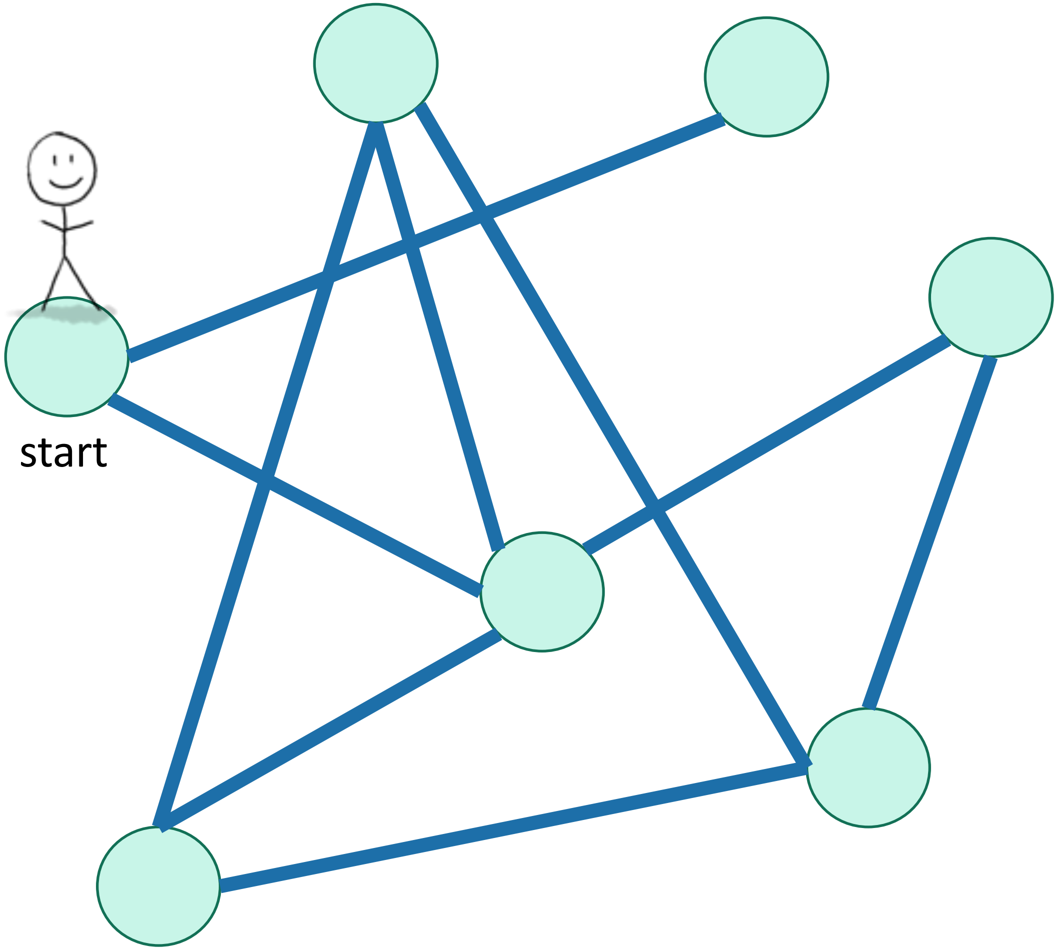
How do we explore a graph?

At each node, you can get a list of neighbors,
and choose to go there if you want.



Depth First Search

Exploring a labyrinth with chalk and a piece of string



● Not been there yet

● Been there, haven't explored all the paths out.

● Been there, have explored all the paths out.

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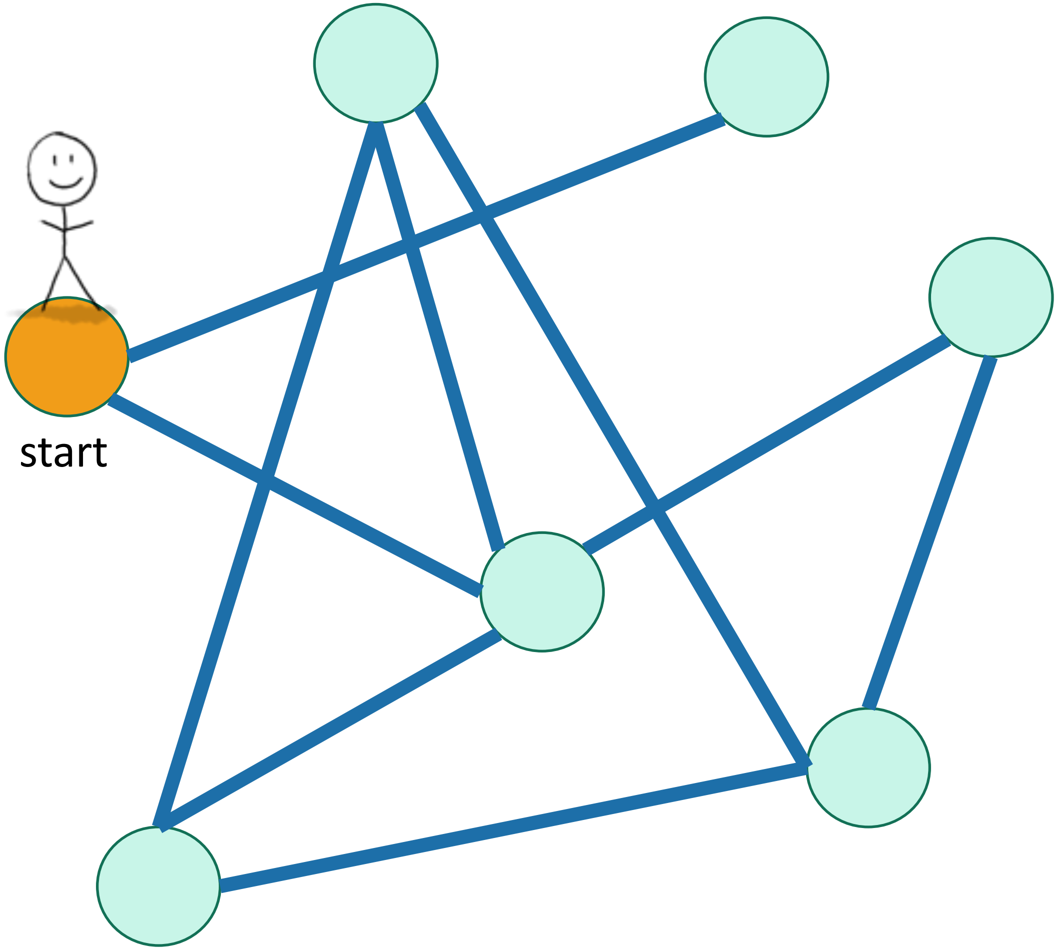

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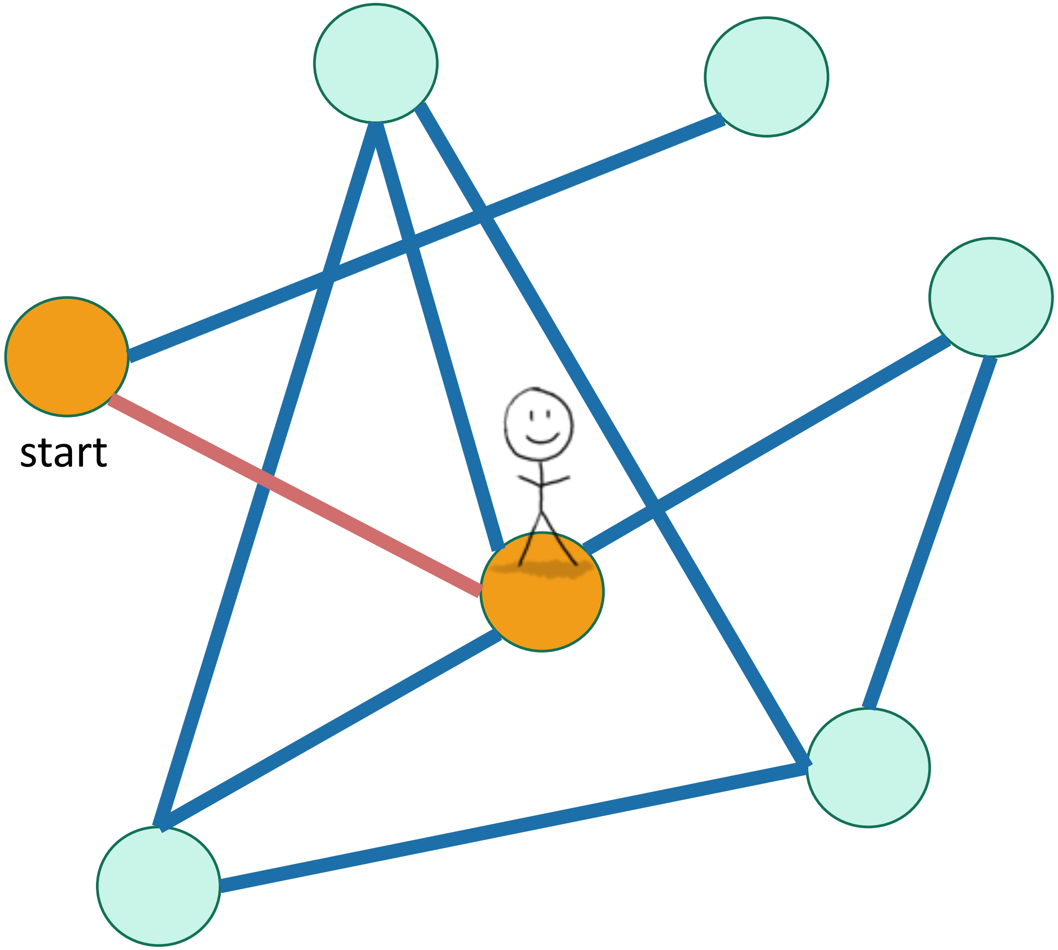

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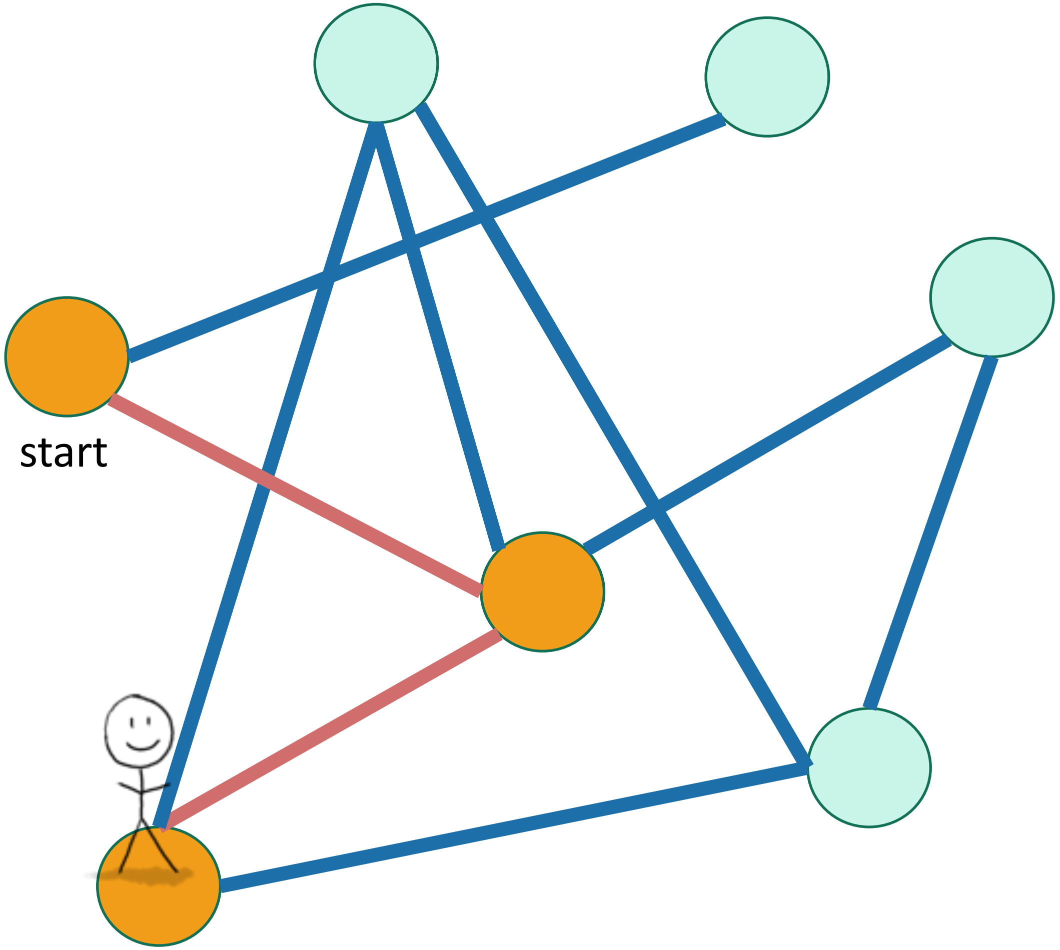

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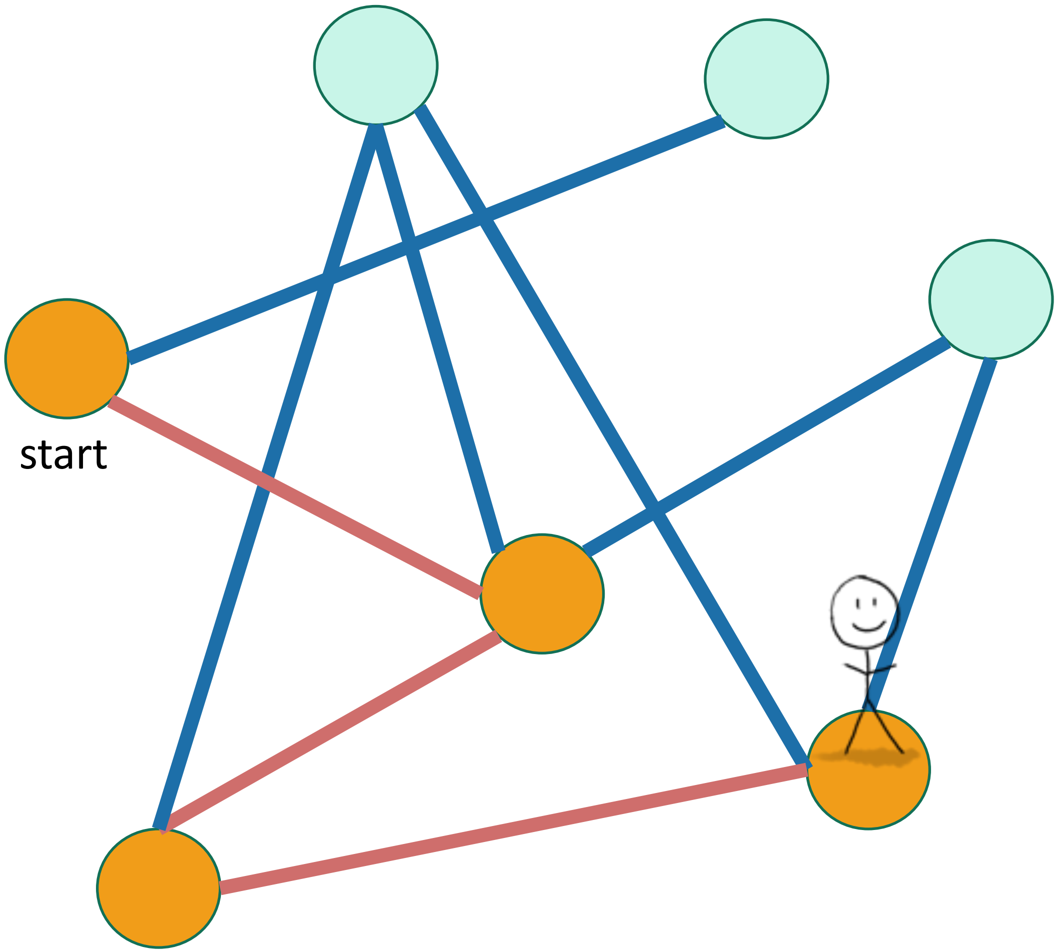

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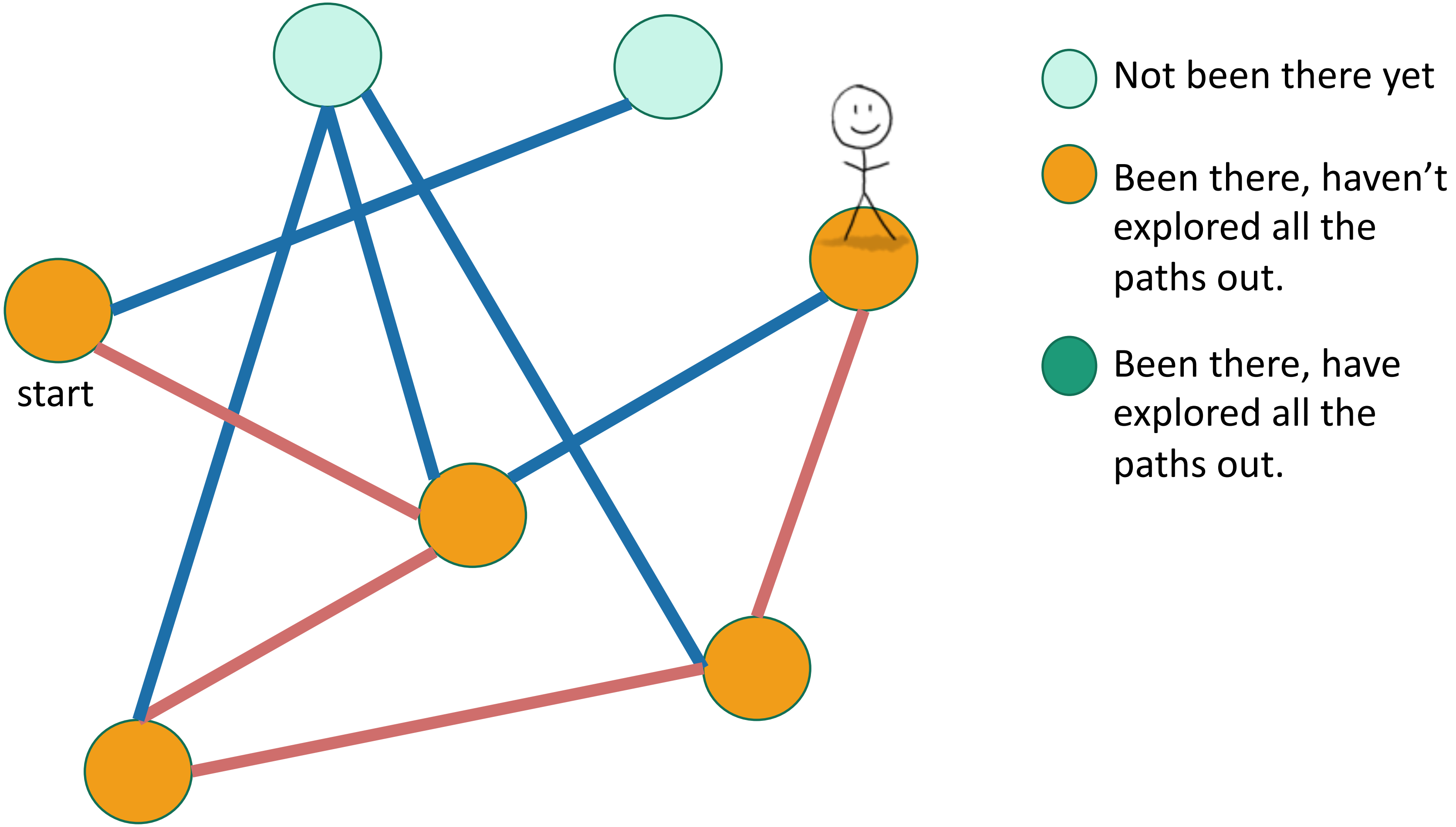

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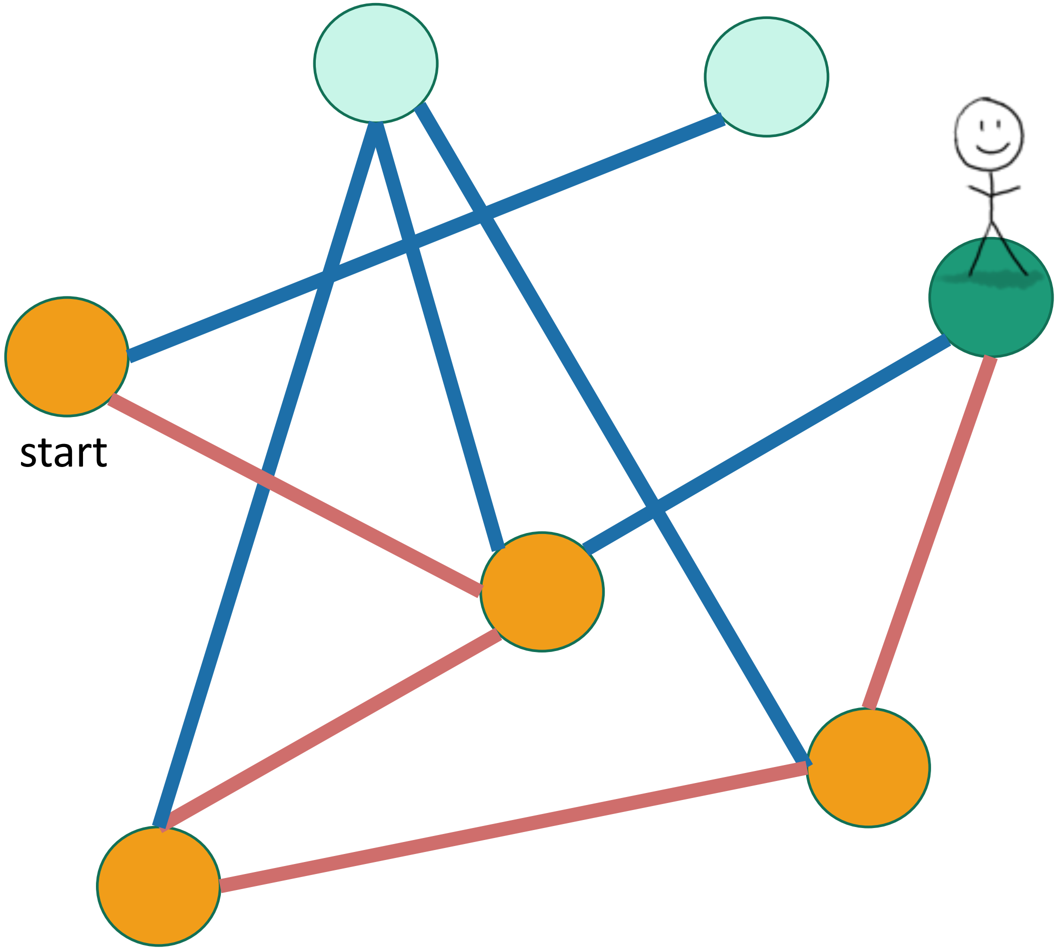

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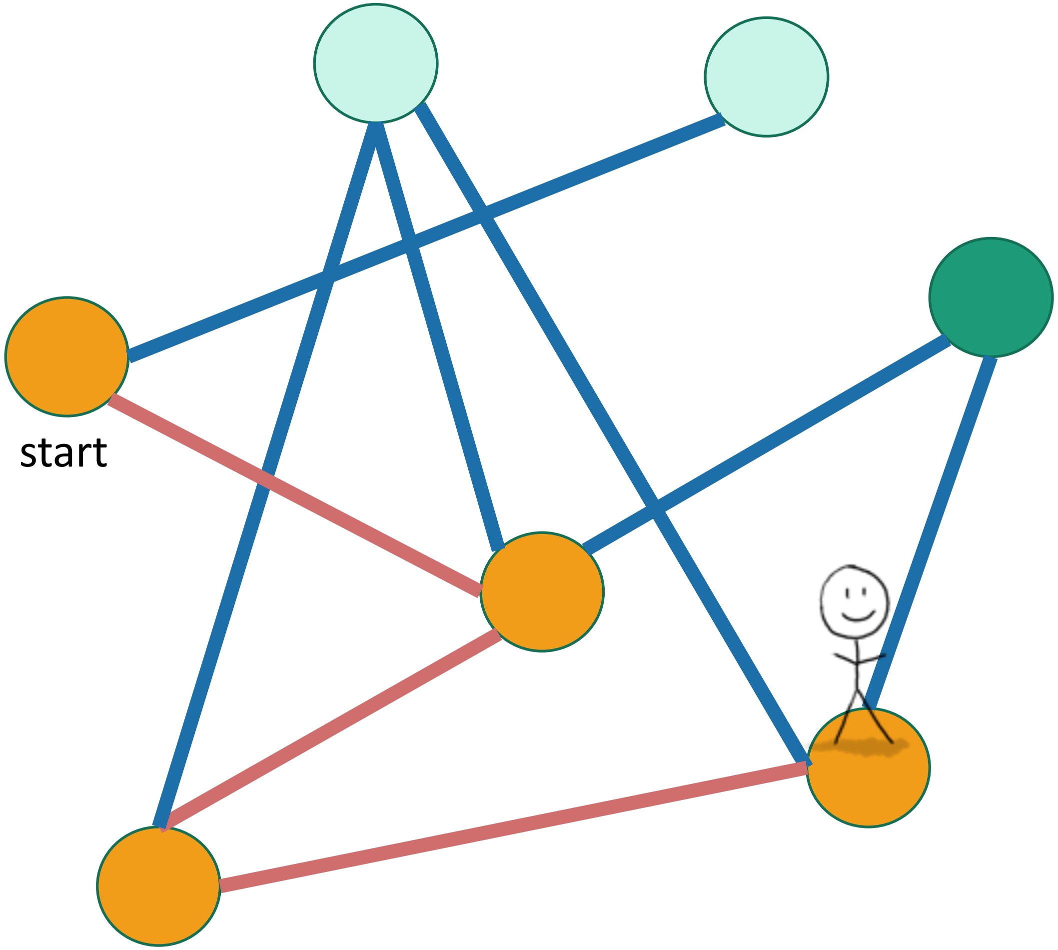

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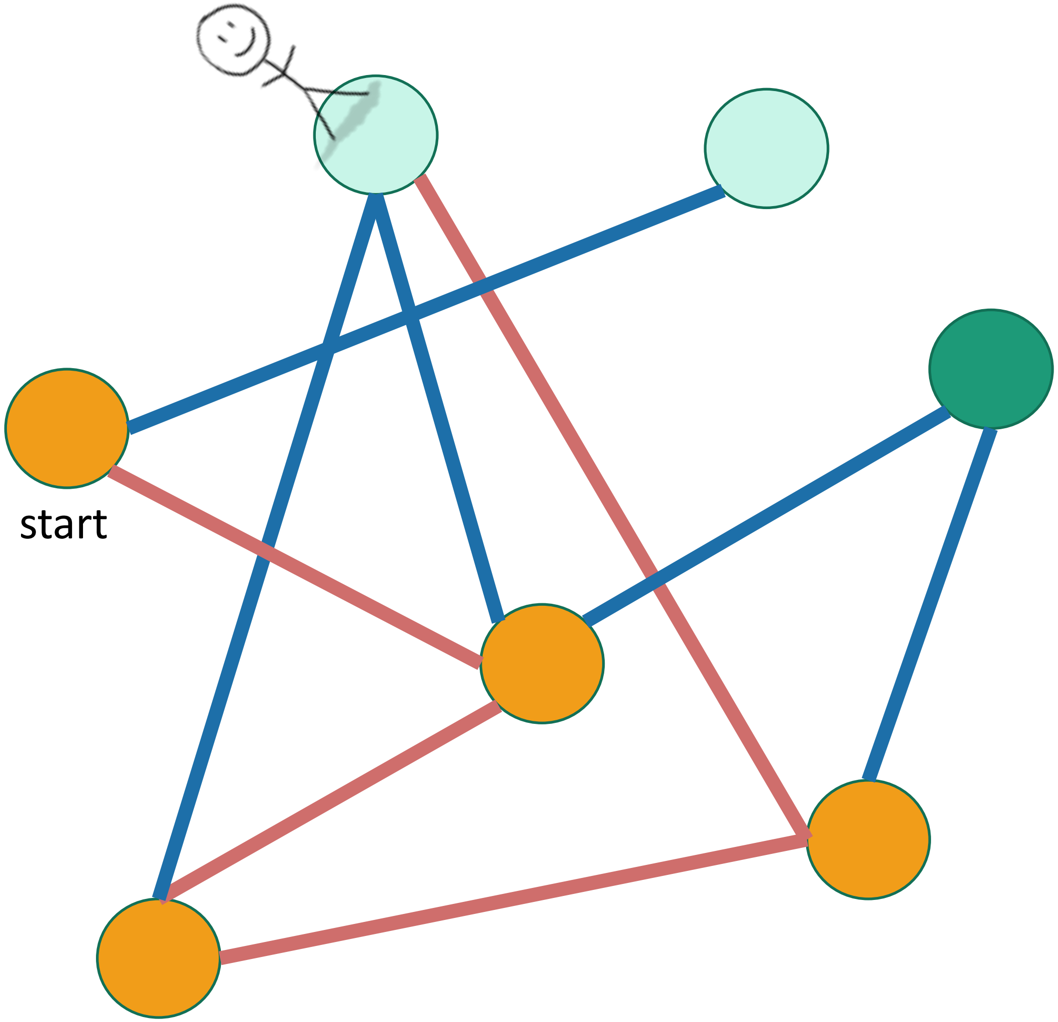

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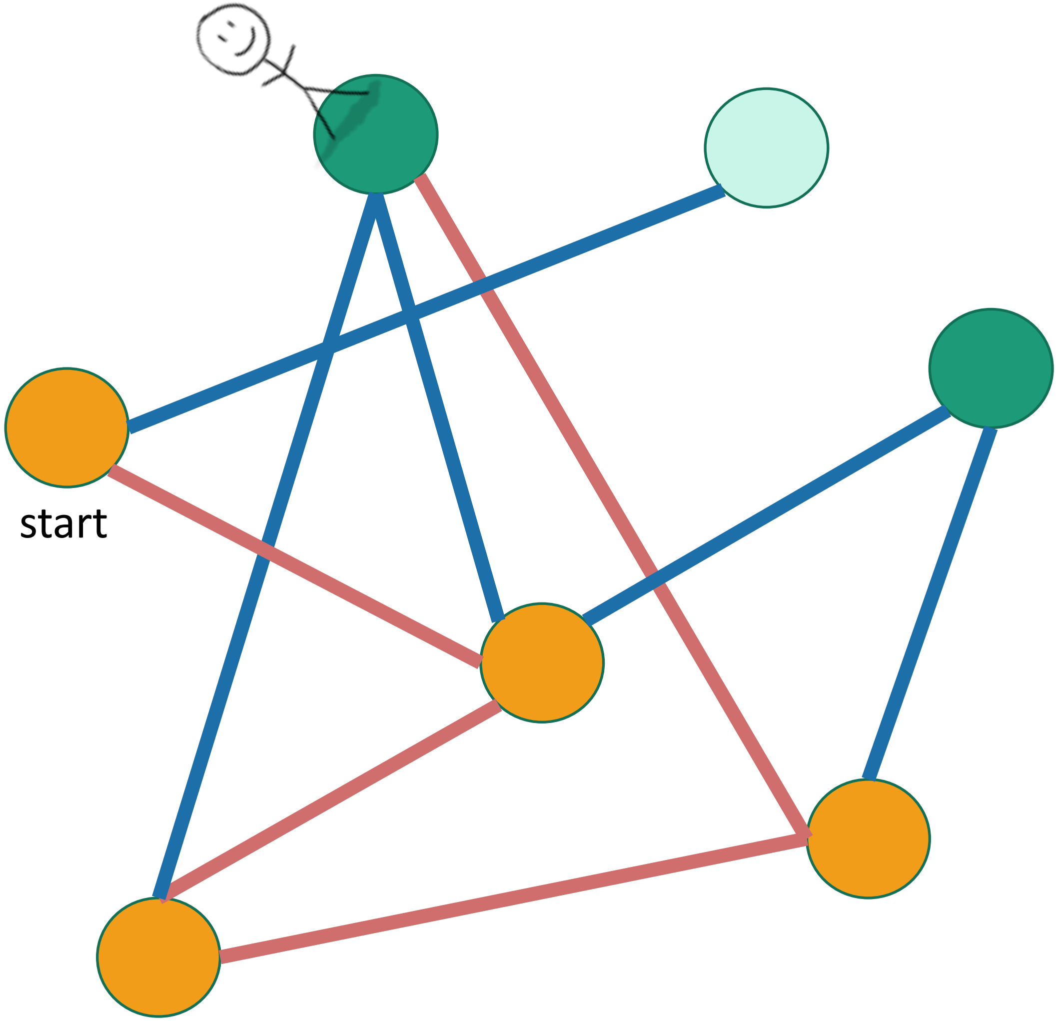

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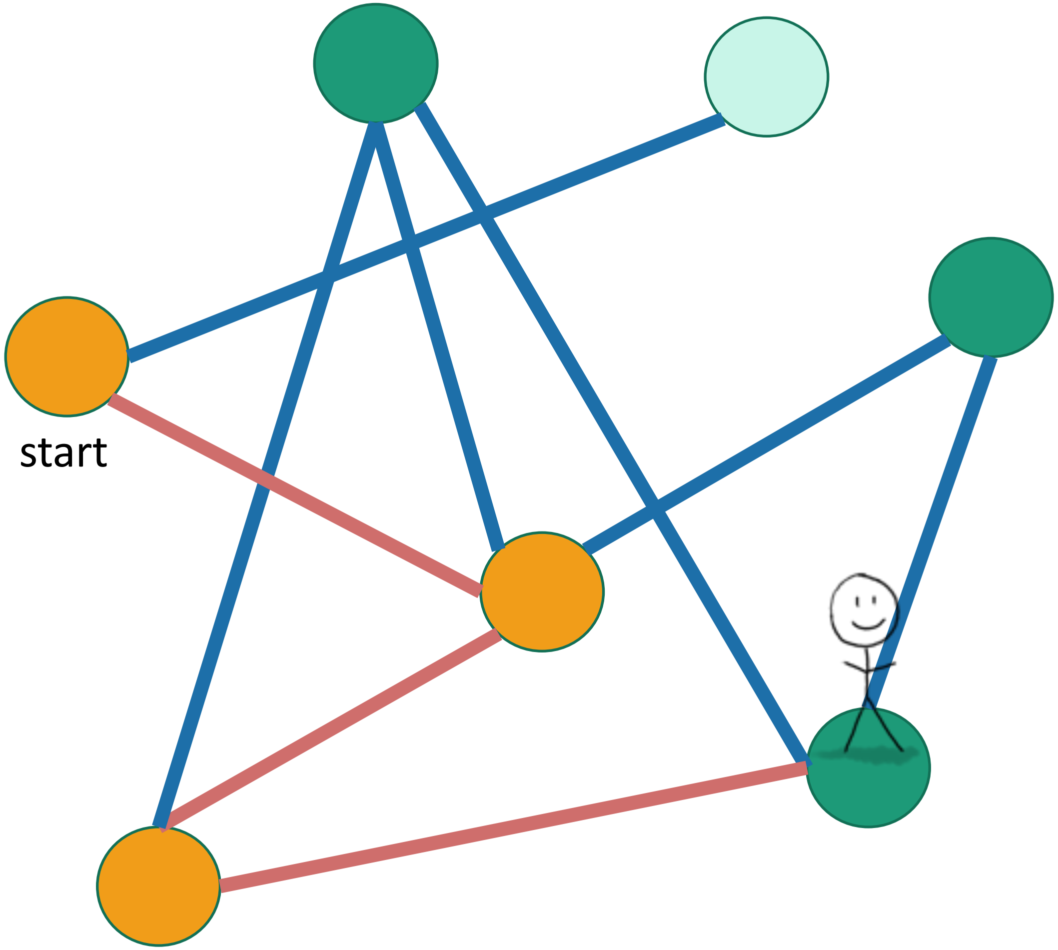

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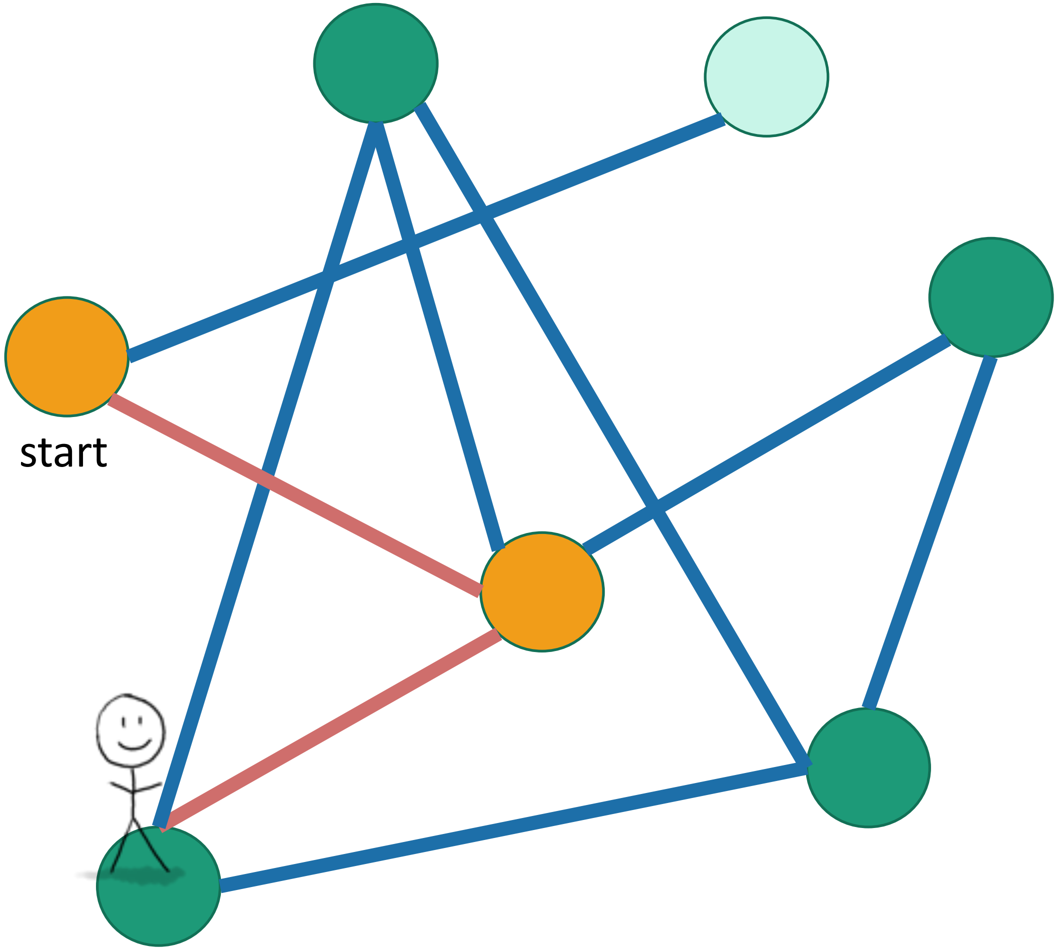

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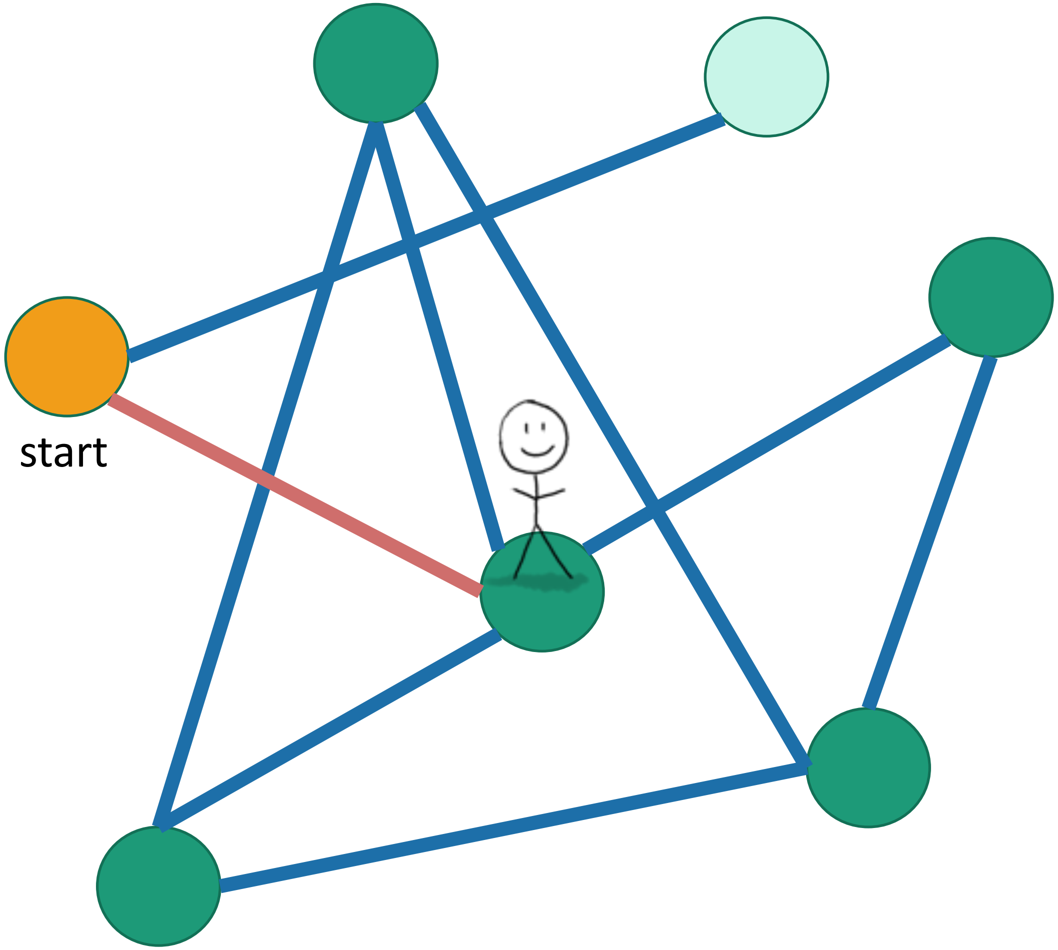

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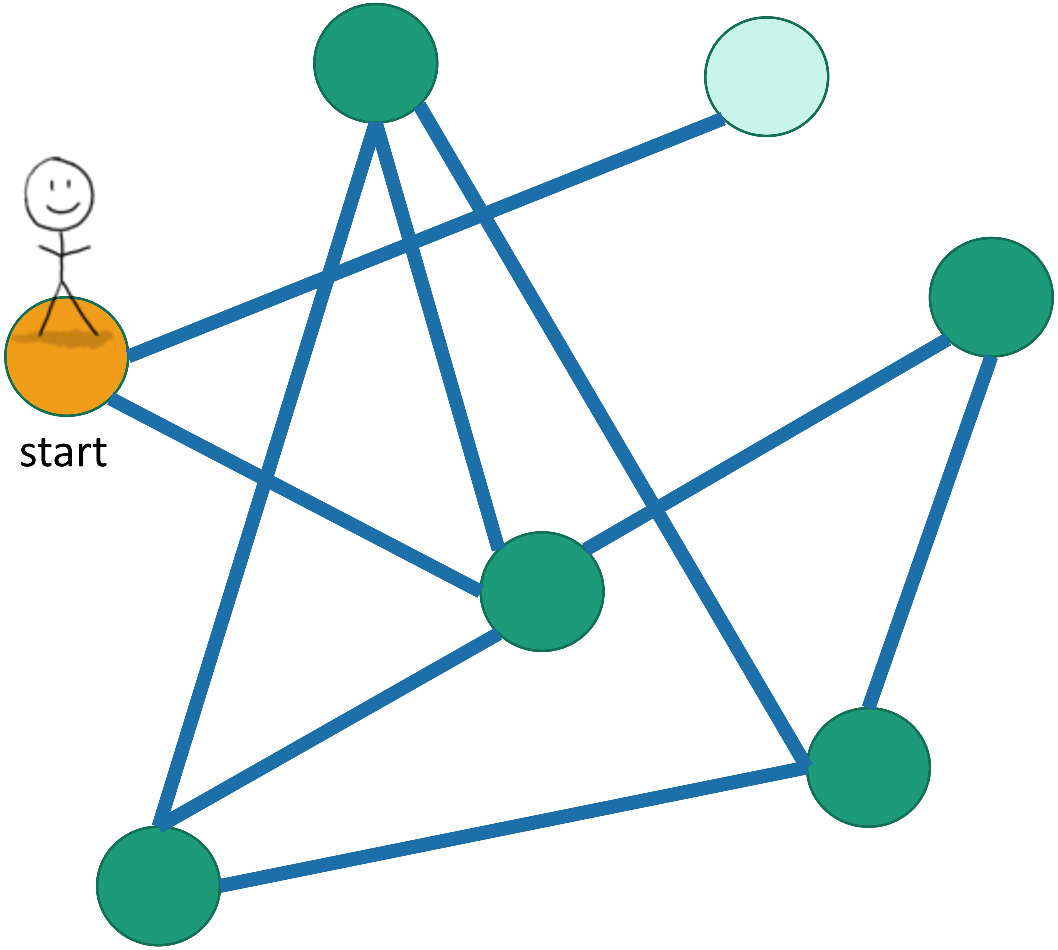

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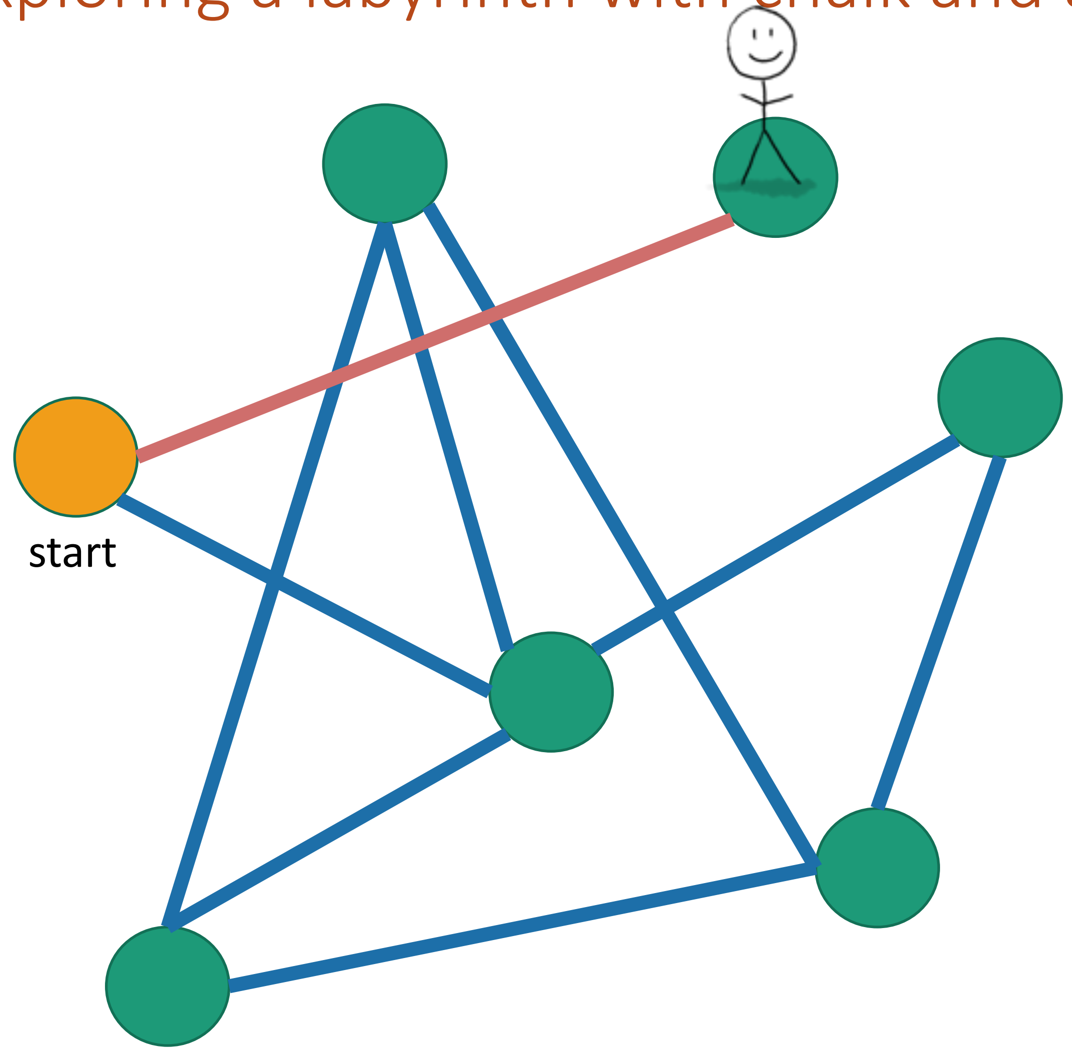

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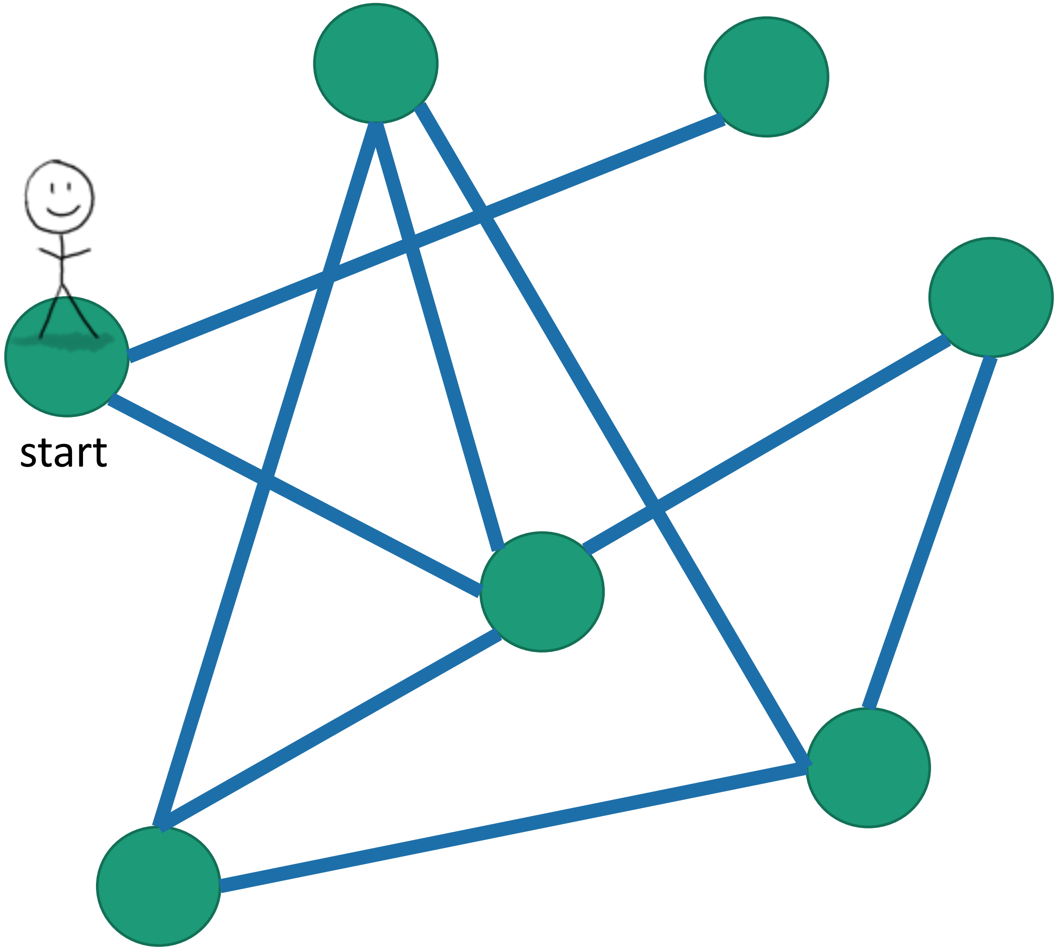

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Labyrinth:

EXPLORED!

Depth First Search

Exploring a labyrinth with pseudocode

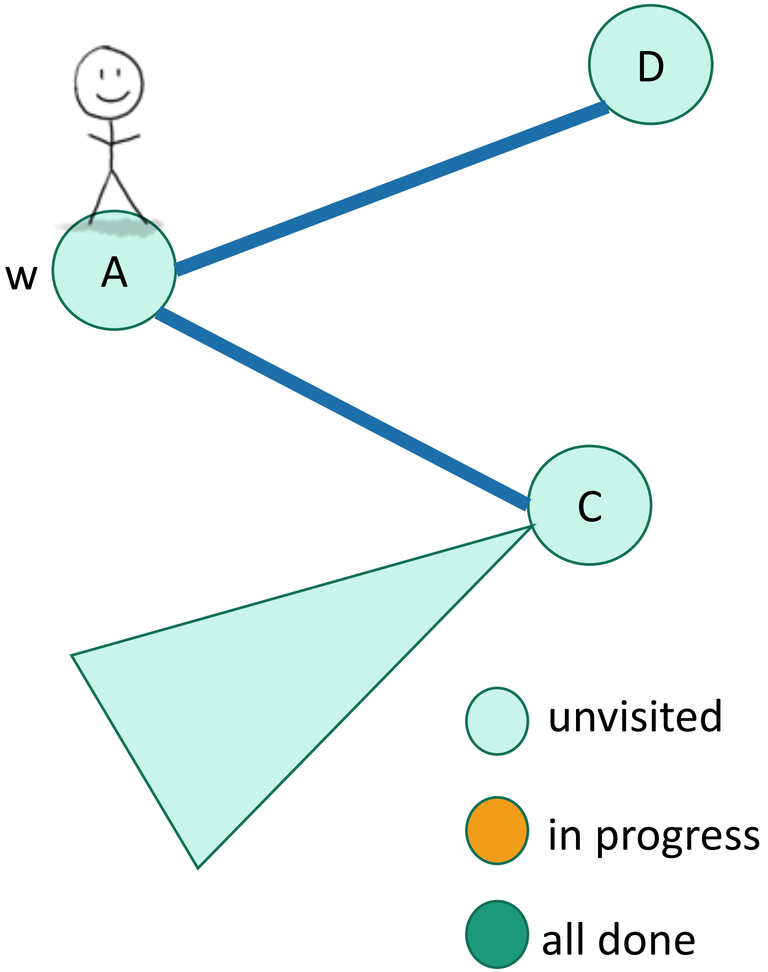
- Each vertex keeps track of whether it is:
  - Unvisited ○
  - In progress 🟠
  - All done 🟢

- Each vertex will also keep track of:
  - The time we **first enter it**.
  - The time we finish with it and mark it **all done**.

You might have seen other ways to implement DFS than what we are about to go through. This way has more bookkeeping – the bookkeeping will be useful later!

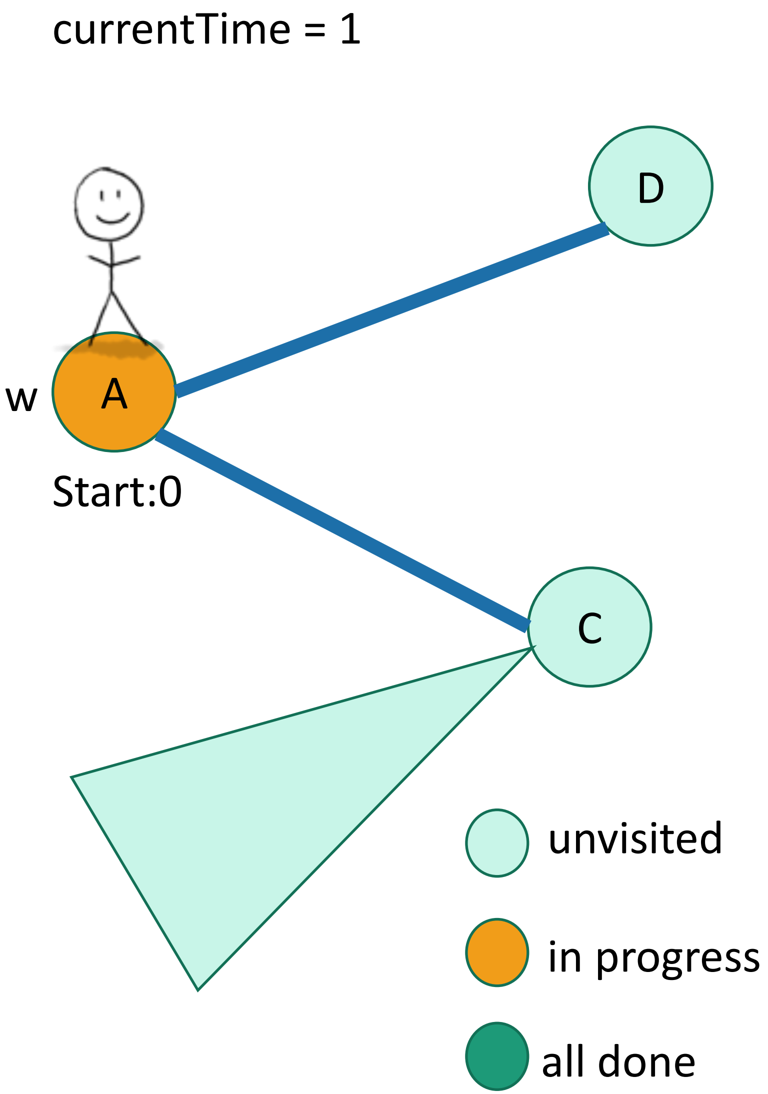
Depth First Search

currentTime = 0



```
• DFS(w, currentTime):
    • w.startTime = currentTime
    • currentTime += 1
    • Mark w as in progress.
    • for v in w.neighbors:
        • if v is unvisited:
            • currentTime
                  = DFS(v, currentTime)
            • currentTime += 1
    • w.finishTime = currentTime
    • Mark w as all done
    • return currentTime
```

Depth First Search



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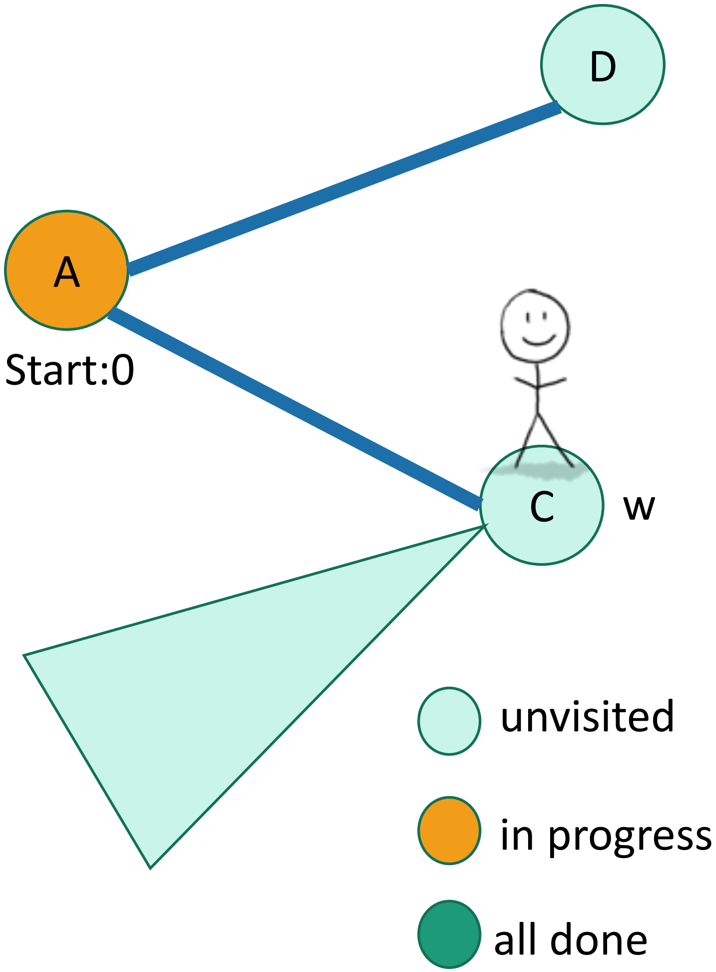
DFS(w, currentTime):
    w.startTime = currentTime
    currentTime += 1
    Mark w as in progress.
    for v in w.neighbors:
        if v is unvisited:
            currentTime
                = DFS(v, currentTime)
            currentTime += 1
    w.finishTime = currentTime
    Mark w as all done
    return currentTime
```

Depth First Search

currentTime = 1



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            currentTime += 1
    w.finishTime = currentTime
    Mark w as all done
    return currentTime
```
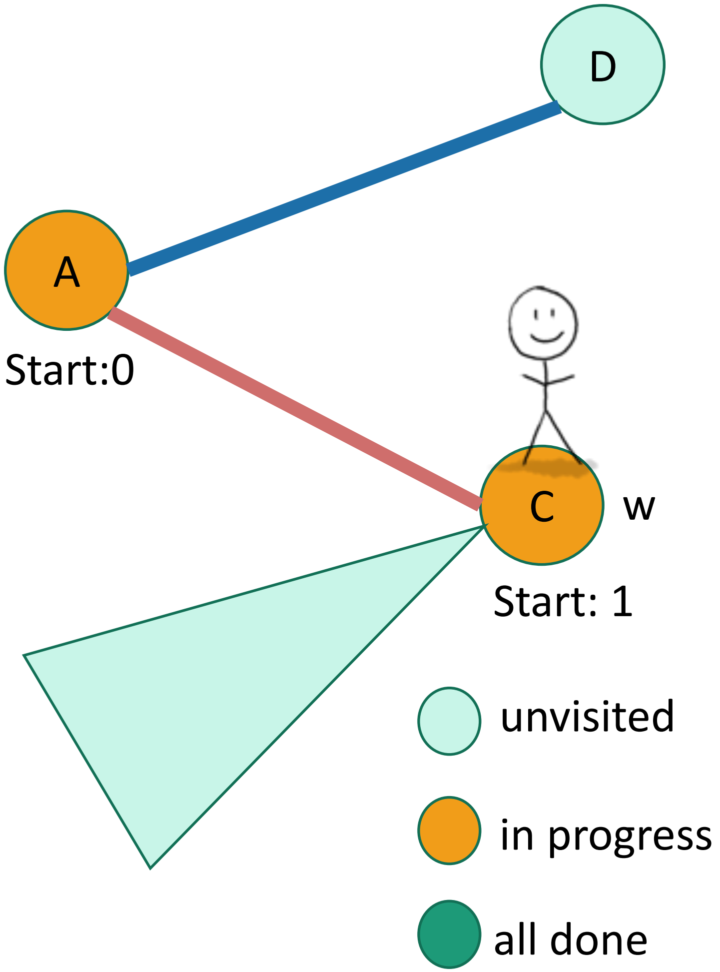

Depth First Search

currentTime = 2



```
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    • w.startTime = currentTime
    • currentTime += 1
    • Mark w as in progress.
    • for v in w.neighbors:
        • if v is unvisited:
            • currentTime
                  = DFS(v, currentTime)
            • currentTime += 1
    • w.finishTime = currentTime
    • Mark w as all done
    • return currentTime
```
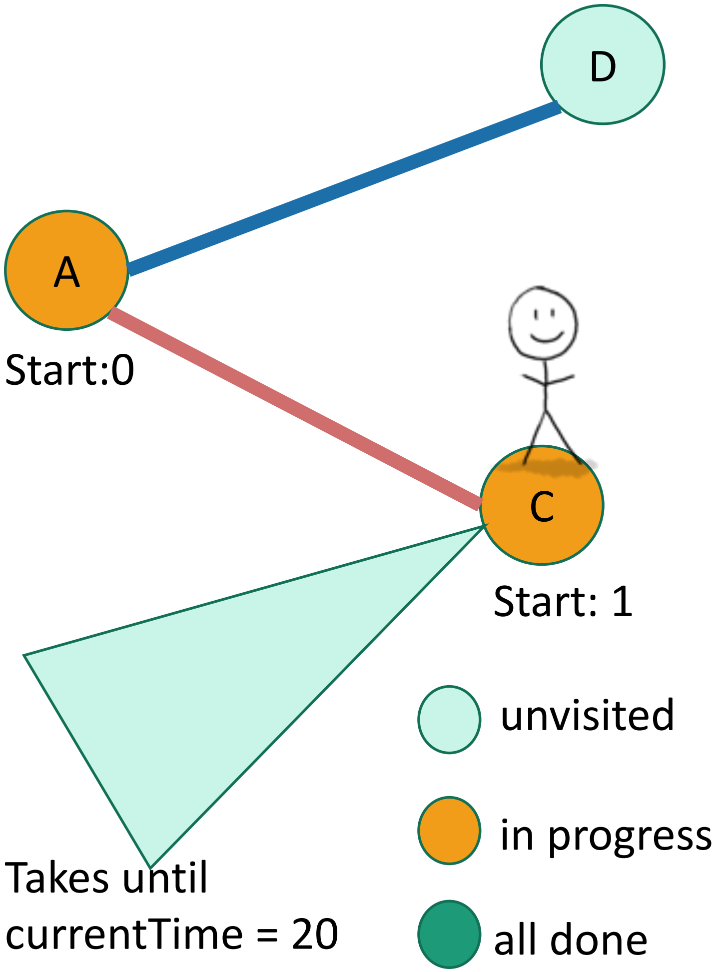

Depth First Search

currentTime = 20



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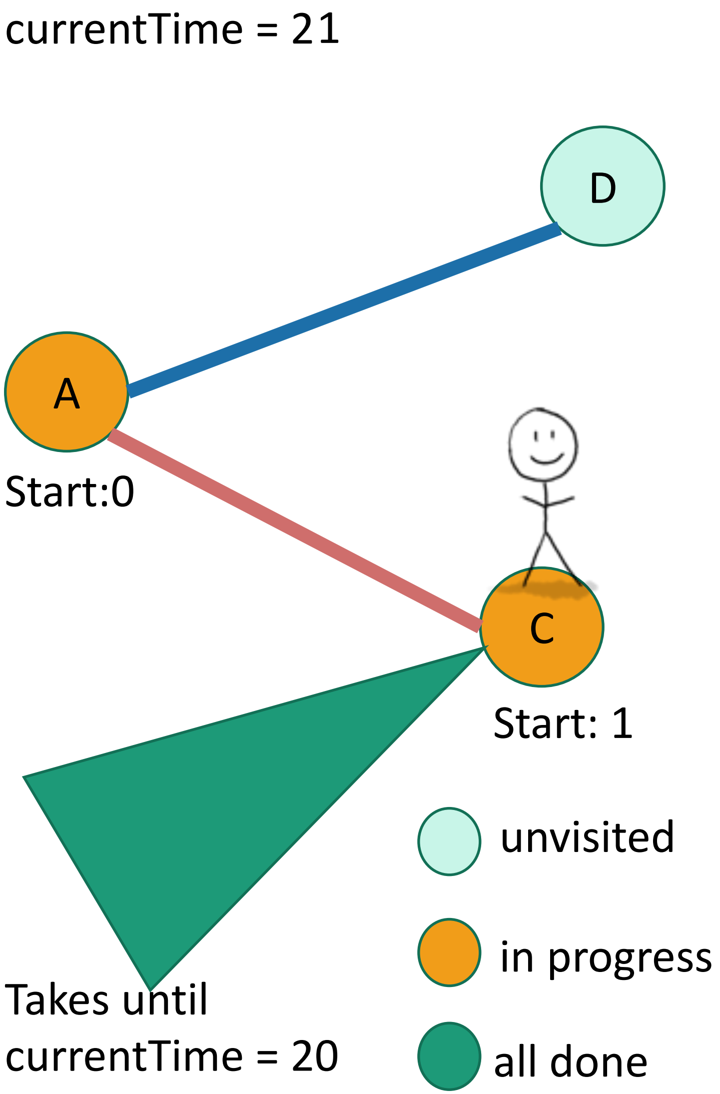
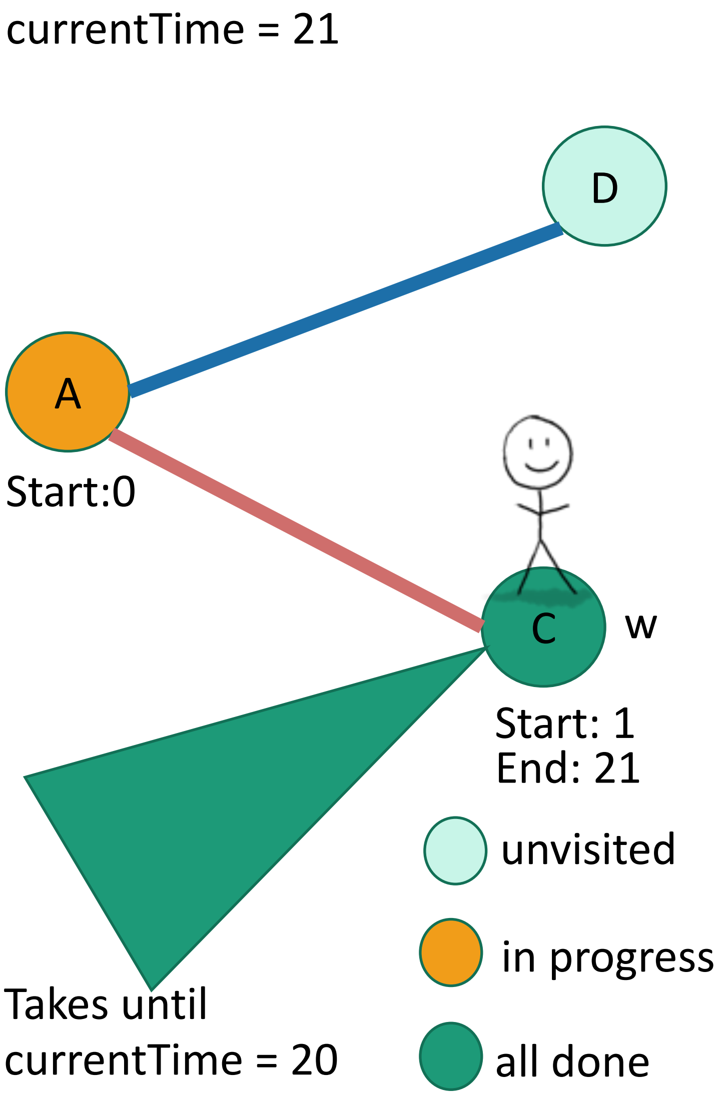
    - currentTime += 1
    - Mark w as in progress.
    - for v in w.neighbors:
        - if v is unvisited:
            - currentTime
                  = DFS(v, currentTime)
            - currentTime += 1
    - w.finishTime = currentTime
    - Mark w as all done
    - return currentTime
```

Depth First Search



```
DFS(w, currentTime):
    w.startTime = currentTime
    currentTime += 1
    Mark w as in progress.
    for v in w.neighbors:
        if v is unvisited:
            currentTime
                = DFS(v, currentTime)
            currentTime += 1
    w.finishTime = currentTime
    Mark w as all done
    return currentTime
```

Depth First Search



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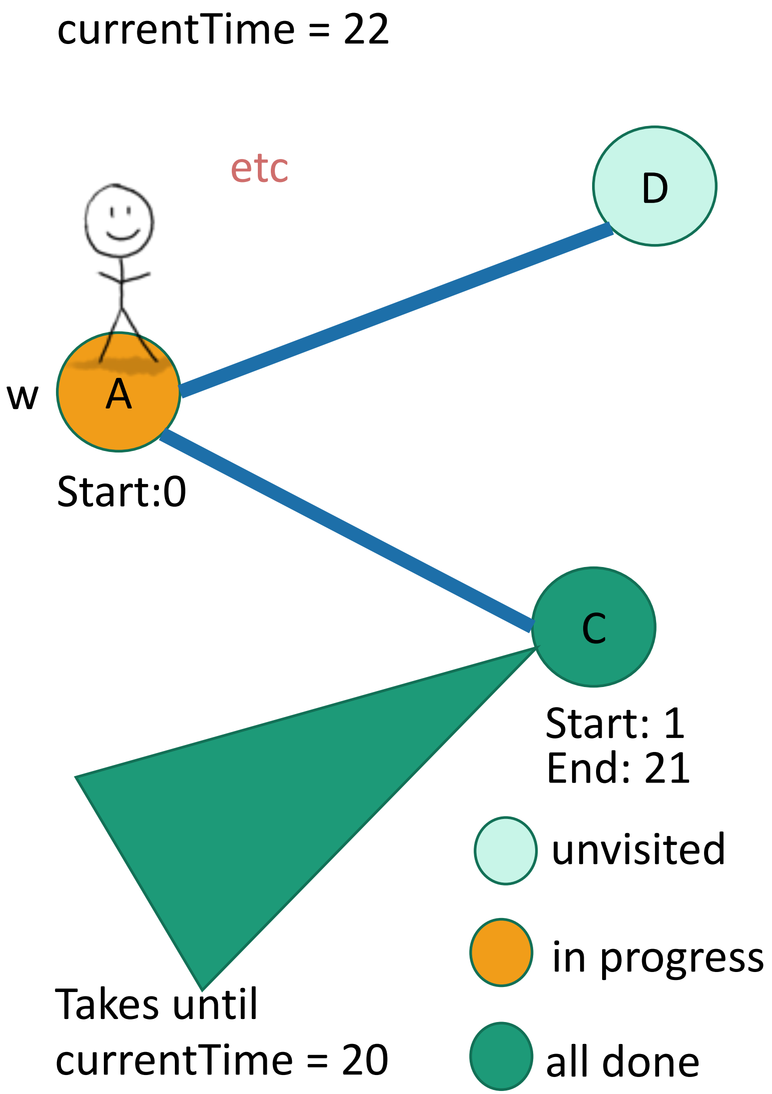
Depth First Search



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            currentTime
                = DFS(v, currentTime)
            currentTime += 1
    w.finishTime = currentTime
    Mark w as all done
    return currentTime
```
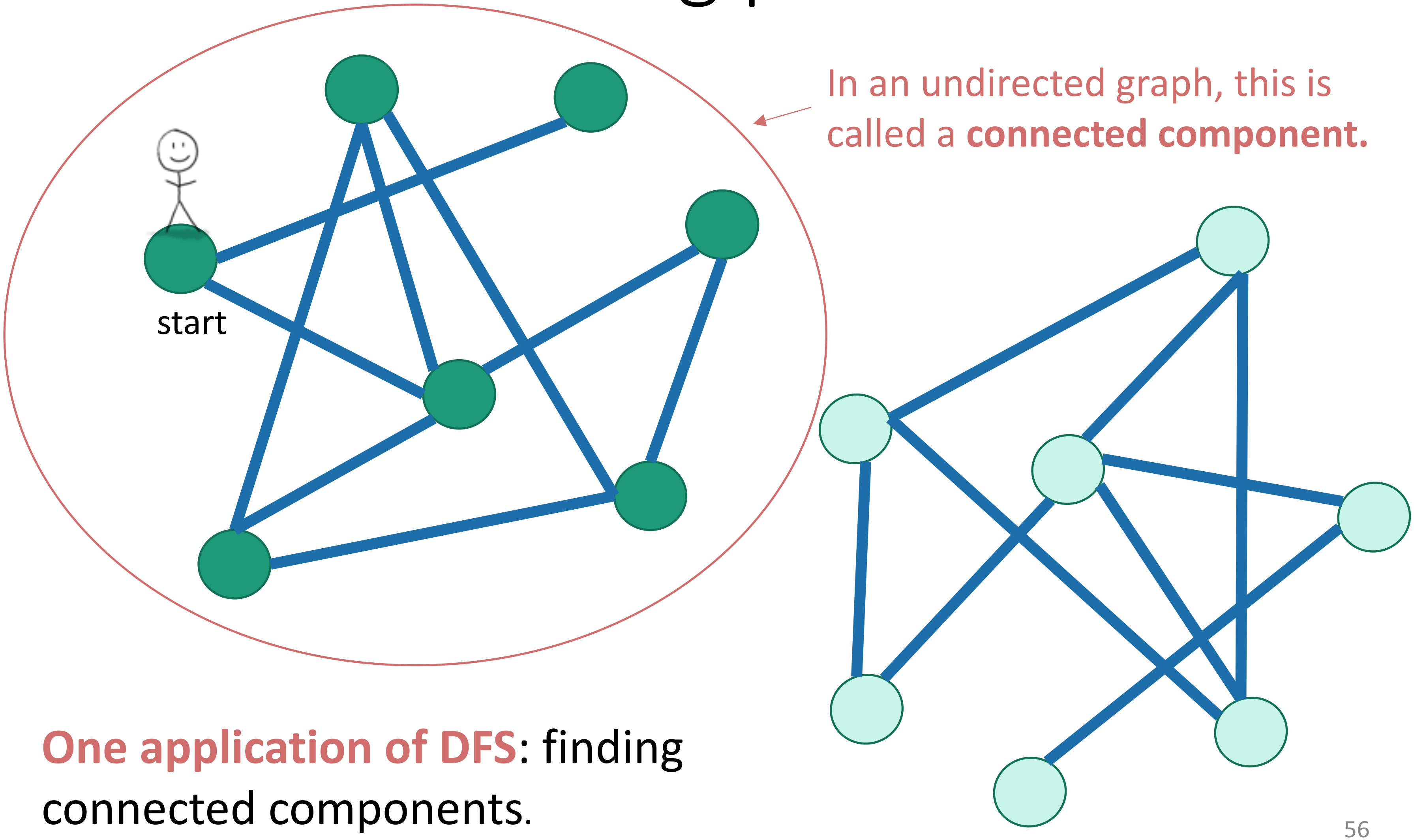

This is not the only way to write DFS!

- See the lecture notes for an iterative version (using stacks)! If your graph is large and stack overflow a concern, use this version.
- (Or figure out how to do it yourself!)

DFS finds all the nodes reachable from the starting point

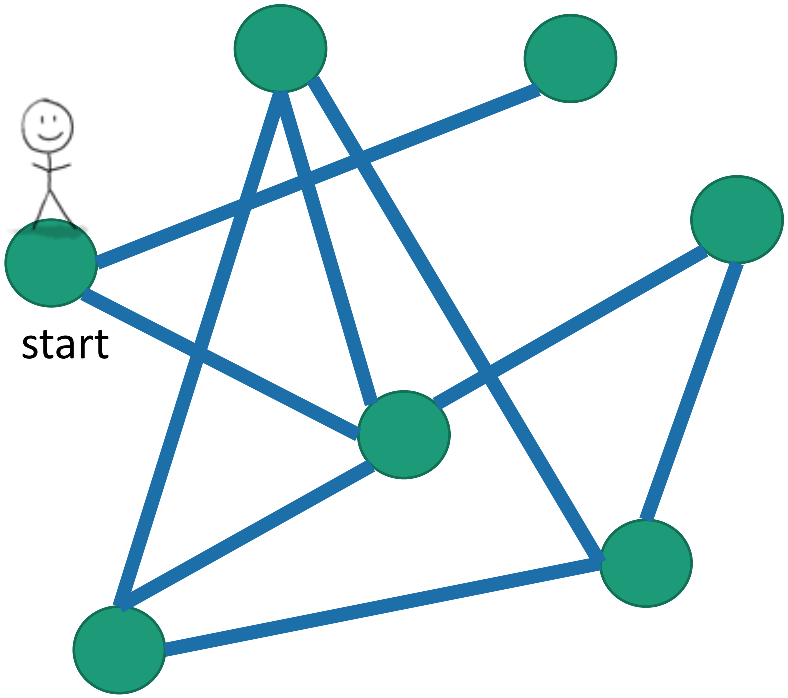


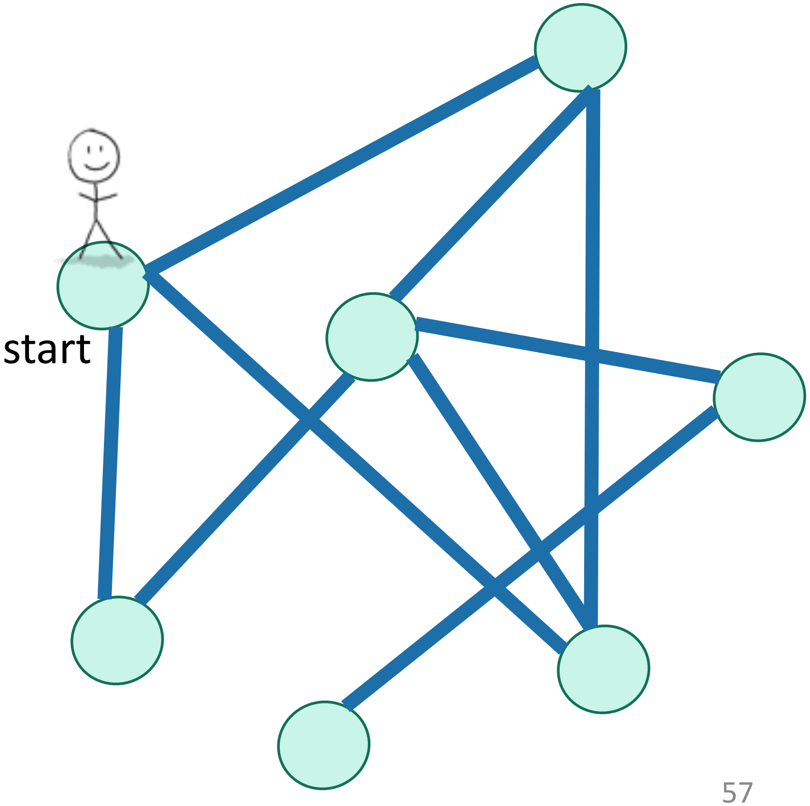
In an undirected graph, this is called a **connected component.**

One application of DFS: finding connected components.

To explore the whole graph

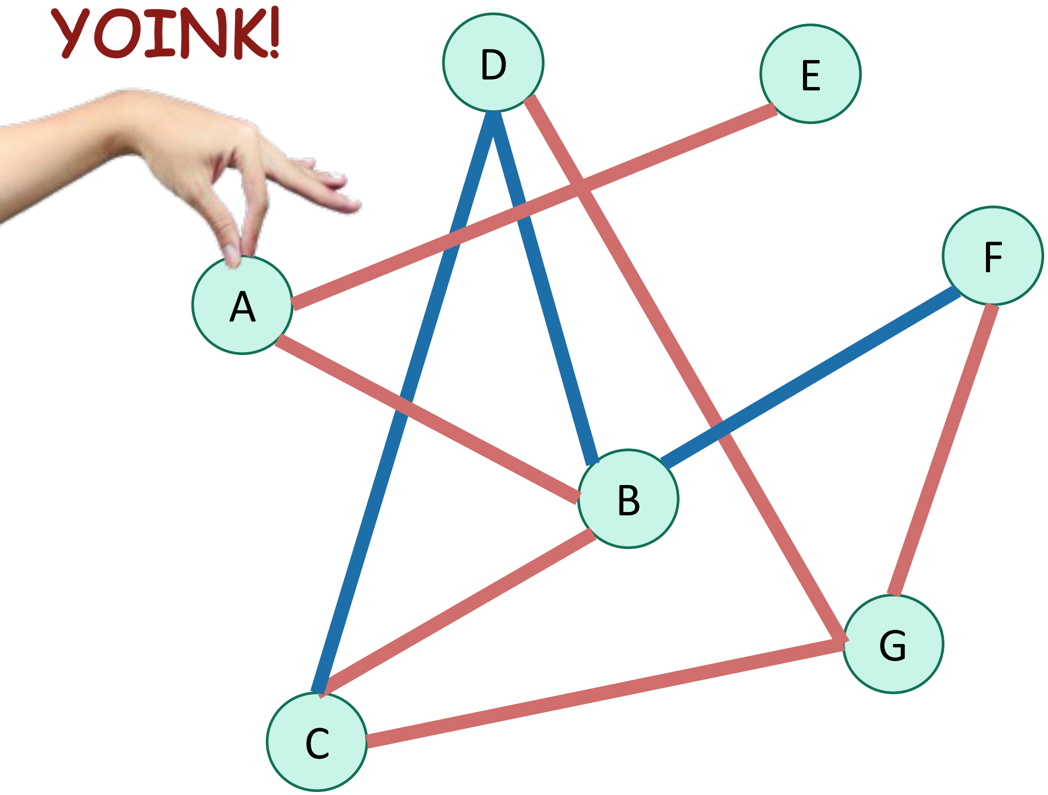
- Do it repeatedly!



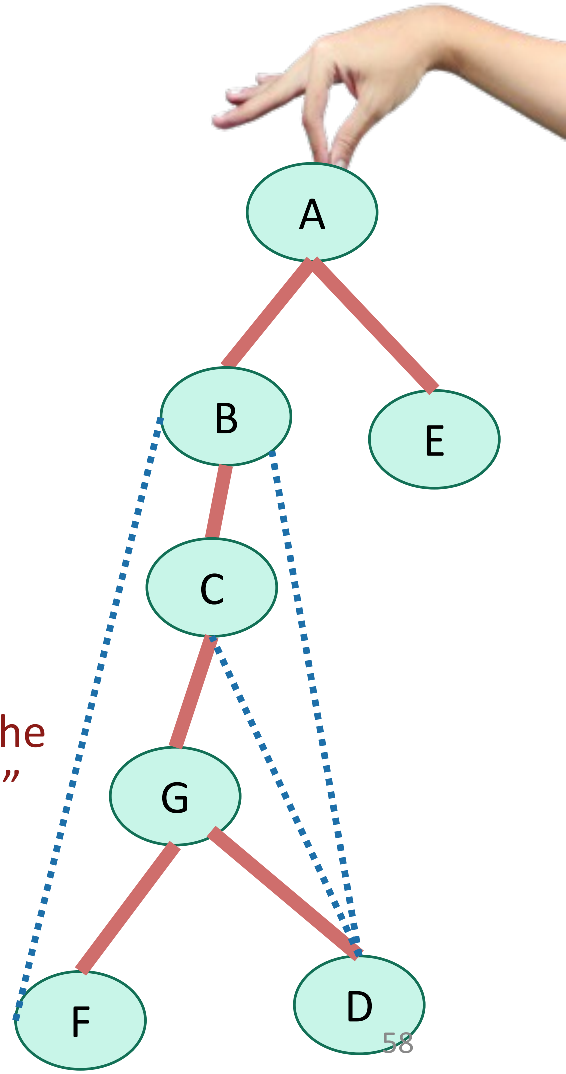


Why is it called depth-first?

- We are implicitly building a tree:



Call this the “DFS tree”



- First, we go as deep as we can.

Running time

To explore **just the connected component** we started in

- We look at each edge at most twice.
  - Once from each of its endpoints
- And basically, we don't do anything else.
- So…

$$O(m)$$

Running time

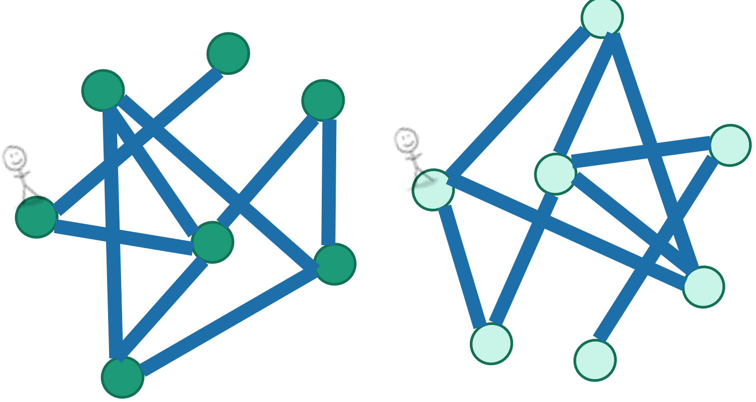
To explore just the connected component we started in

- Assume we are using the linked-list format for G.
- Say C = (V', E') is a connected component.
- We visit each vertex in V' exactly once.
  - Here, "visit" means "call DFS on"
- At each vertex w, we:
  - Do some book-keeping: O(1)
  - Loop over w's neighbors and check if they are visited (and then potentially make a recursive call): O(1) per neighbor or O(deg(w)) total.
- Total time:
  - $\sum_{w \in V'} (O(\deg(w)) + O(1))$
  - $= O(|E'| + |V'|)$
  - $= O(|E'|)$

In a connected graph, $|V'| \leq |E'| + 1$.

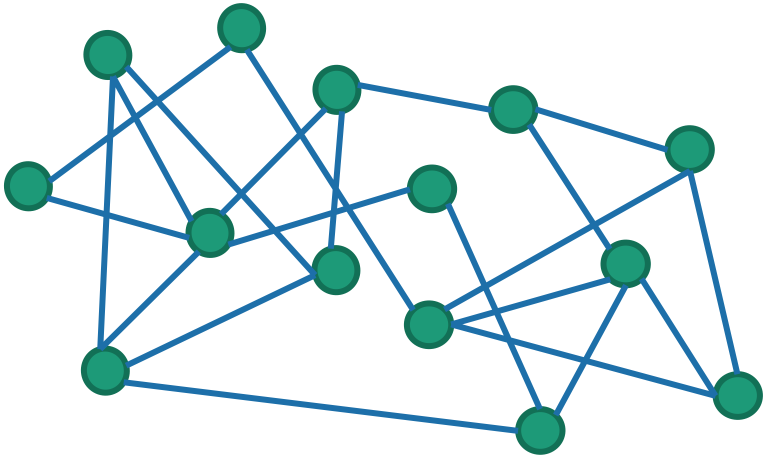
Running time

To explore the whole graph



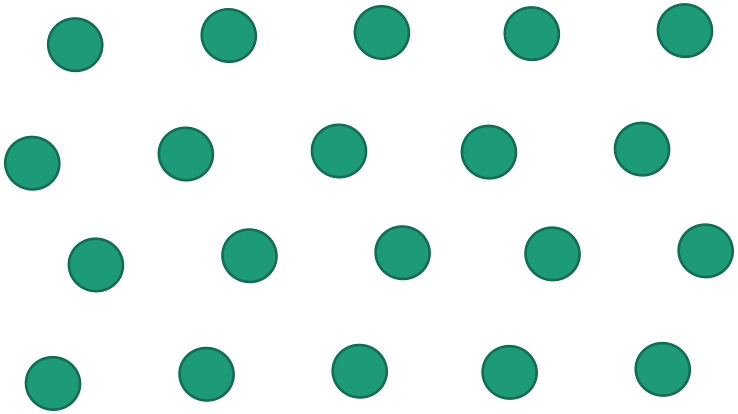
- Explore the connected components one-by-one.
- This takes time O(n + m)
  - Same computation as before:

$$\sum_{w \in V} (O(\deg(w)) + O(1)) = O(|E| + |V|) = O(n + m)$$



Here the running time is O(m) like before

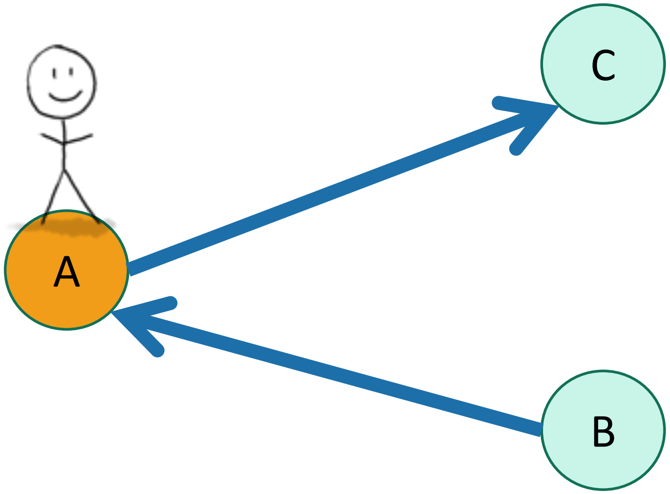
or



Here m=0 but it still takes time O(n) to explore the graph.

You check:

DFS works fine on directed graphs too!



Only walk to C, not to B.

Siggi the studious stork

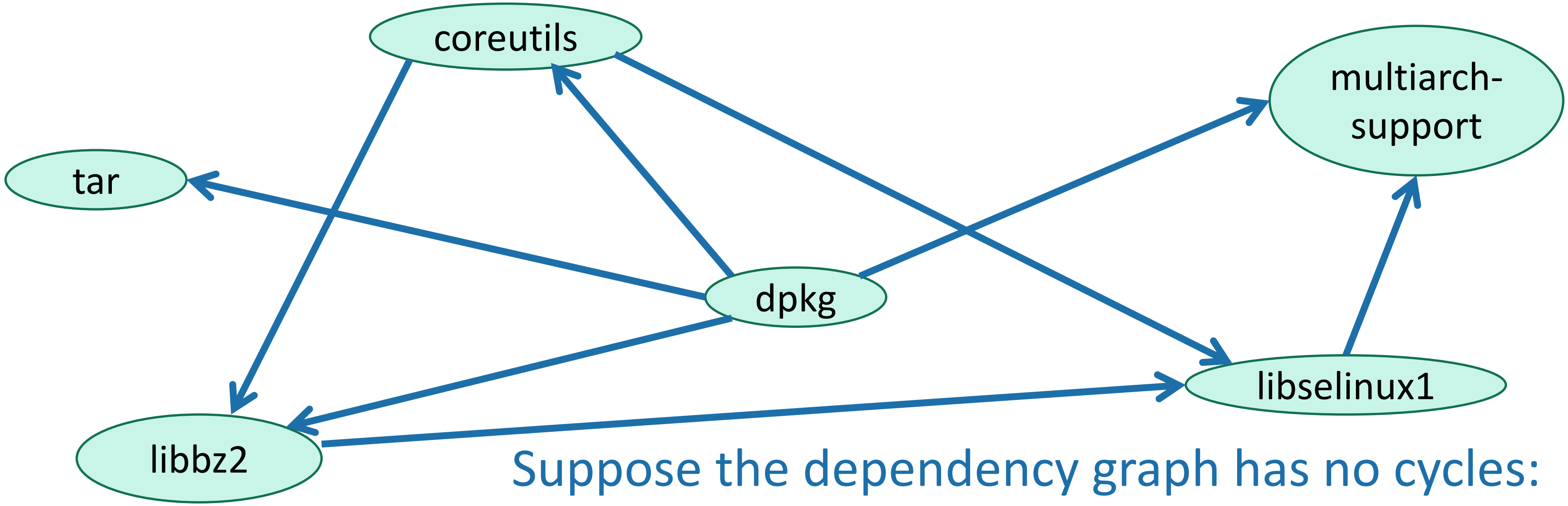
Pre-lecture exercise

- How can you sign up for classes so that you never violate the pre-req requirements?

- More practically, how can you install packages without violating dependency requirements?

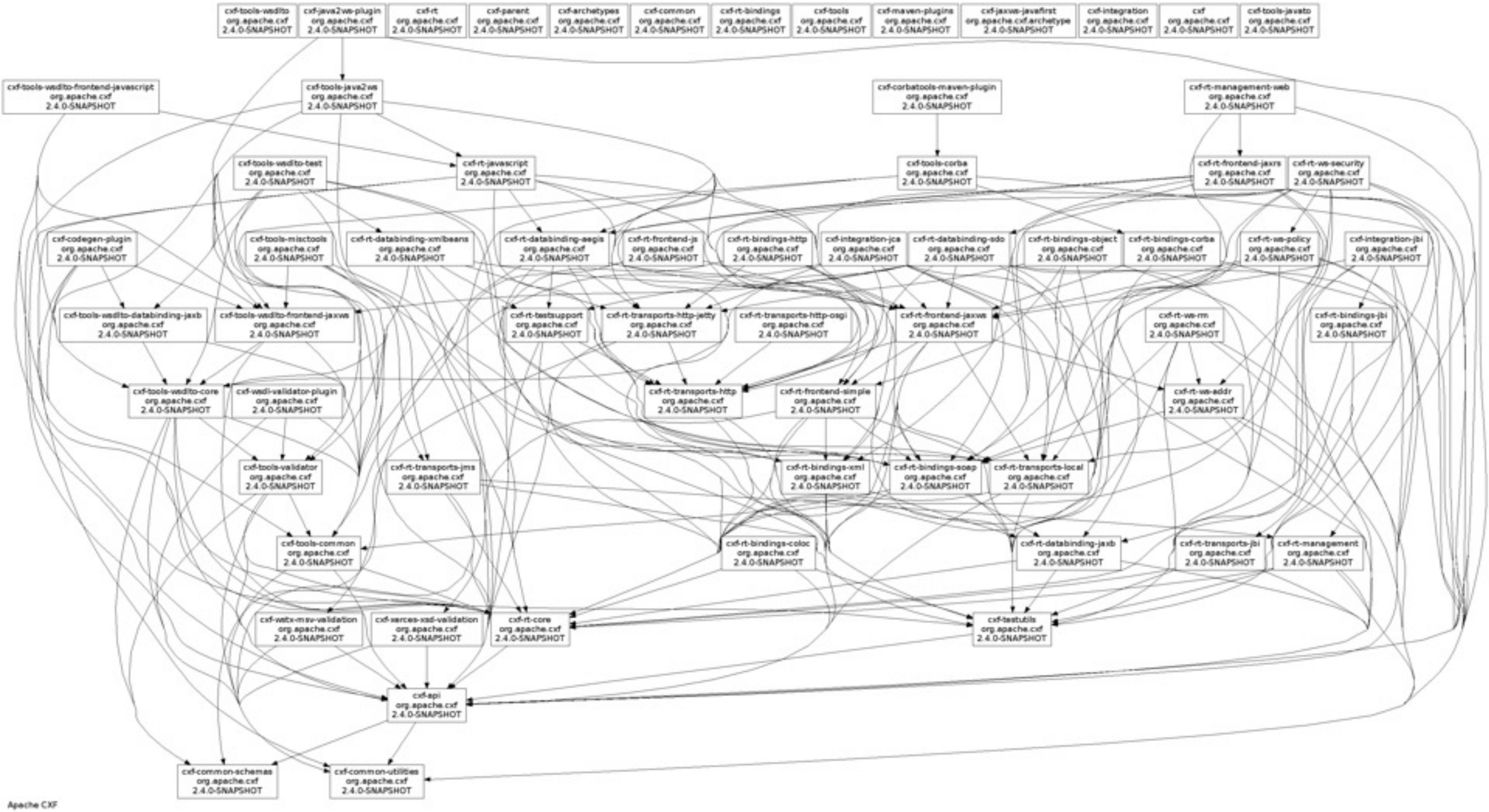
Application of DFS: topological sorting

- Find an ordering of vertices so that all of the dependency requirements are met.
  - Aka, if v comes before w in the ordering, there is not an edge from w to v.



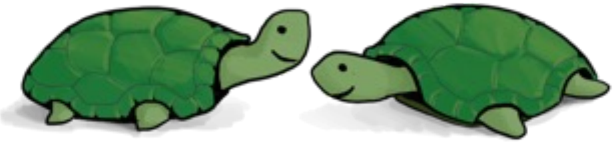
Suppose the dependency graph has no cycles: it is a **Directed Acyclic Graph (DAG)**

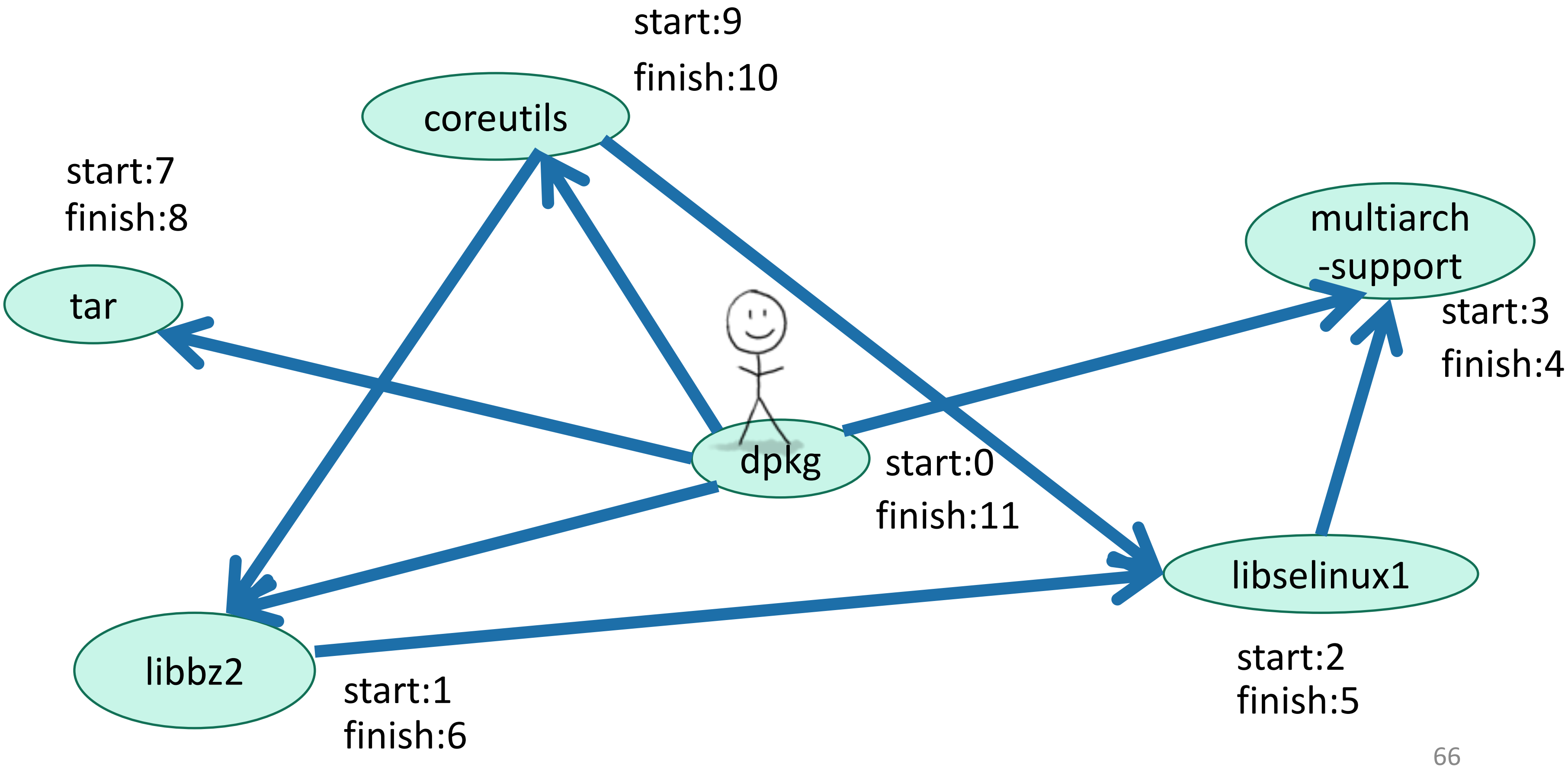
Can’t always eyeball it.



Let’s do DFS

What do you notice about the finish times? Any ideas for how we should do topological sort?
1 minute think
(wait) 1 minute share





Suppose the underlying graph has no cycles

Finish times seem useful

Claim: In general, we'll always have:

finish: [larger]

finish: [smaller]

To understand why, let's go back to that DFS tree.

A more general statement

(this holds even if there are cycles)

(check this statement carefully!)

- If v is a descendant of w in this tree:

timeline

w.start v.start v.finish w.finish

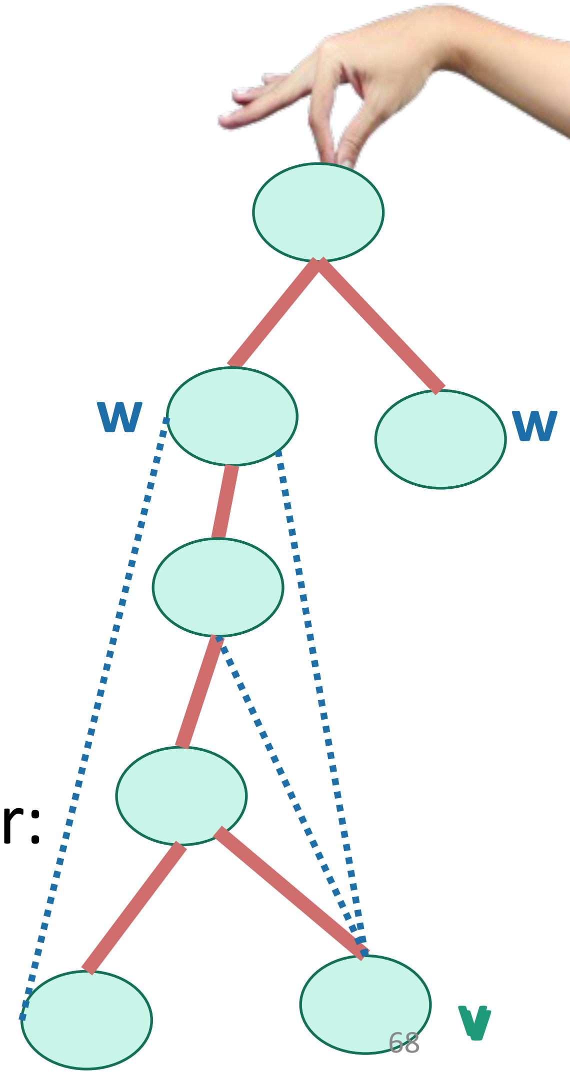
- If w is a descendant of v in this tree:

v.start w.start w.finish v.finish

- If neither are descendants of each other:

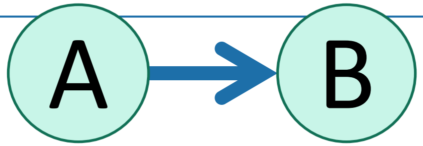
v.start v.finish w.start w.finish

(or the other way around)



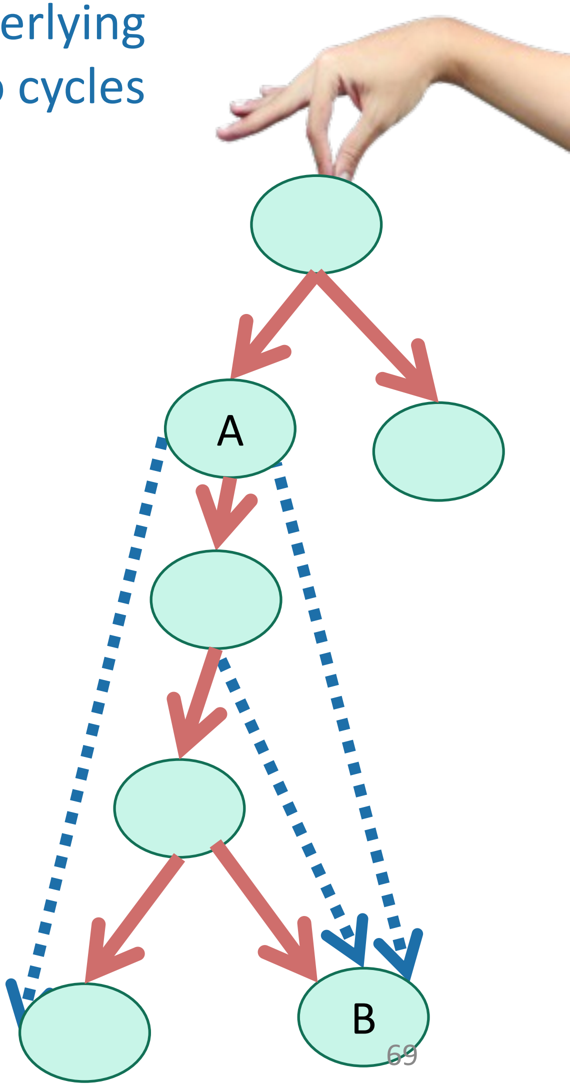
So to prove this →

If



Then B.finishTime < A.finishTime

Suppose the underlying graph has no cycles

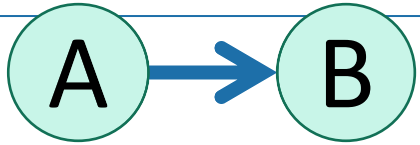


- **Case 1**: B is a descendant of A in the DFS tree.

- Then

- aka, B.finishTime < A.finishTime.

So to prove this →

If


Then B.finishTime < A.finishTime

Suppose the underlying graph has no cycles

- **Case 2**: B is a NOT descendant of A in the DFS tree.
  - Notice that A can't be a descendant of B in the DFS tree or else there'd be a cycle; so it looks like this →
- Then we must have explored B before A.
  - Otherwise we would have gotten to B from A, and B would have been a descendant of A in the DFS tree.
- Then

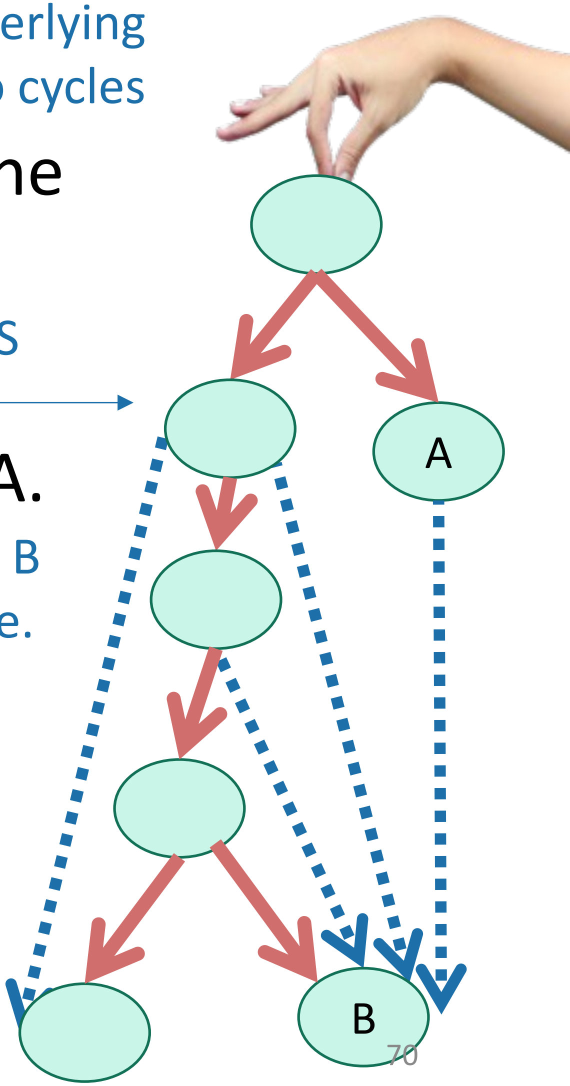

B.finishTime

A.finishTime

B.startTime

A.startTime

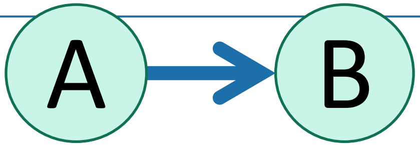
- aka, B.finishTime < A.finishTime.



Theorem

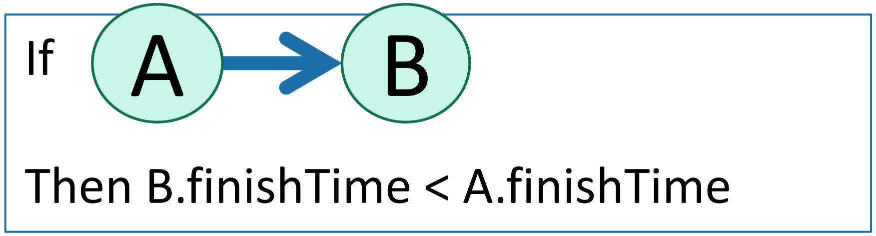
- If we run DFS on a directed acyclic graph,

If



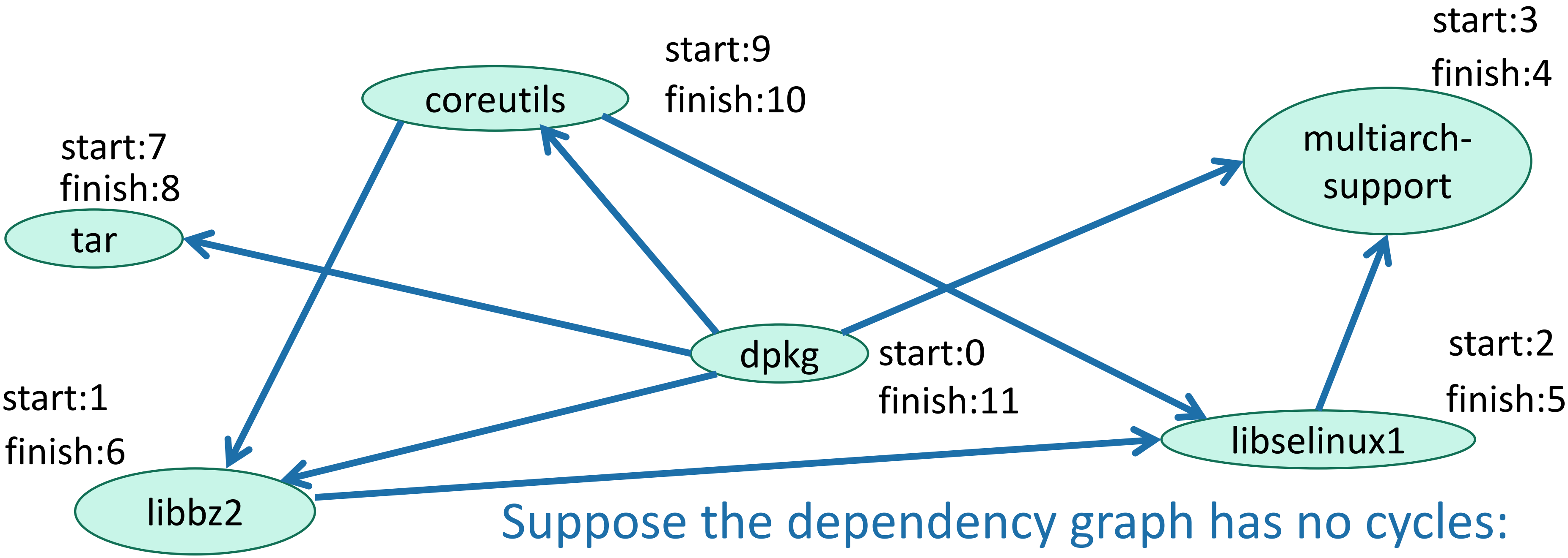
Then B.finishTime < A.finishTime

Back to topological sorting



Then B.finishTime < A.finishTime

- In what order should I install packages?
- In reverse order of finishing time in DFS!

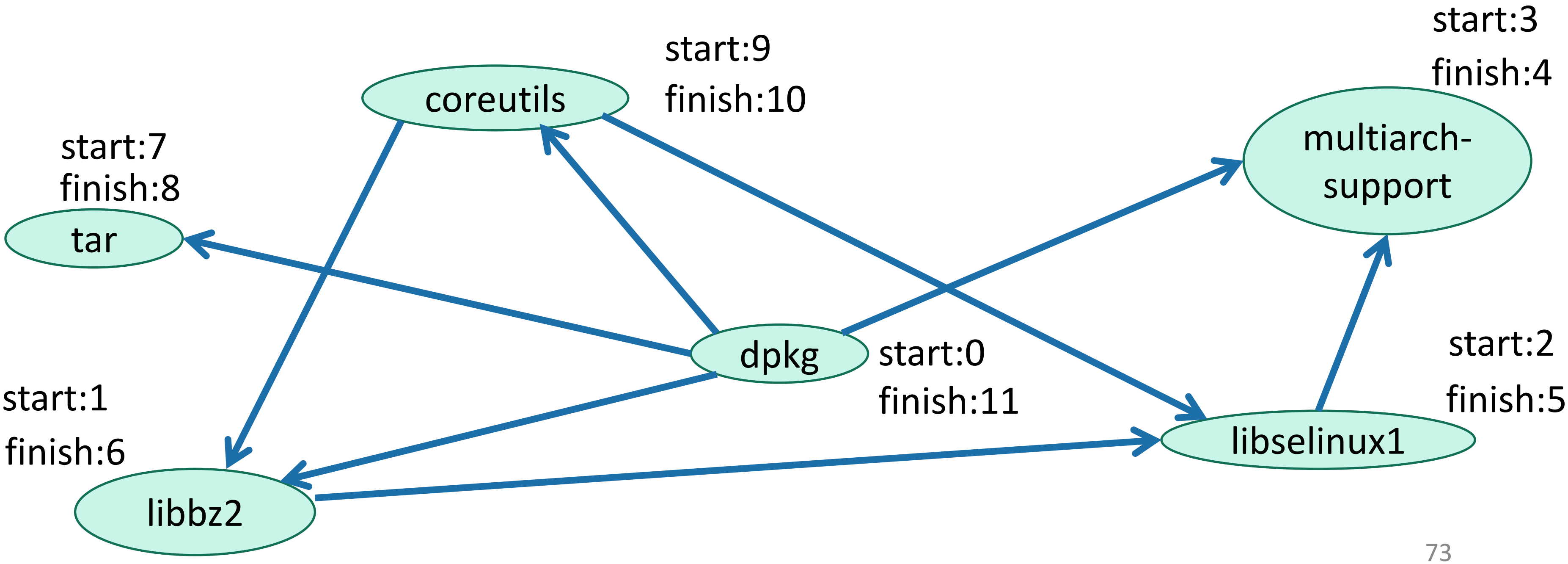


Suppose the dependency graph has no cycles: it is a **Directed Acyclic Graph (DAG)**

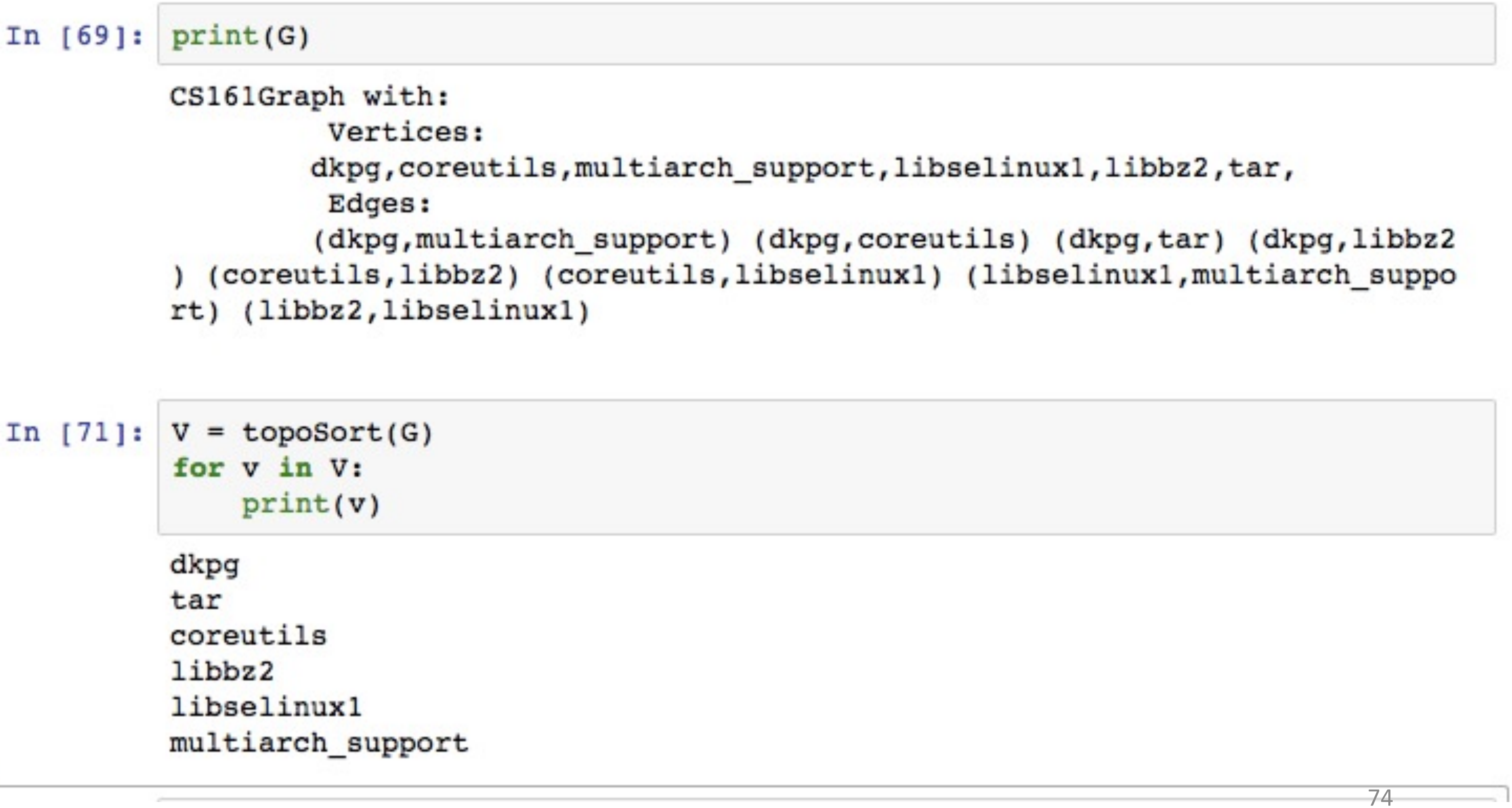
Topological Sorting (on a DAG)

- Do DFS
- When you mark a vertex as **all done**, put it at the **beginning** of the list.

- `dpkg`
- `coreutils`
- `tar`
- `libbz2`
- `libselinux1`
- `multiarch_support`



For implementation,
see Python notebook



In [69]:
```
print(G)
```

```
CS161Graph with:
	 Vertices:
	dkpg,coreutils,multiarch_support,libselinux1,libbz2,tar,
	 Edges:
	(dkpg,multiarch_support) (dkpg,coreutils) (dkpg,tar) (dkpg,libbz2
) (coreutils,libbz2) (coreutils,libselinux1) (libselinux1,multiarch_suppo
rt) (libbz2,libselinux1)
```

In [71]:
```
V = topoSort(G)
for v in V:
    print(v)
```

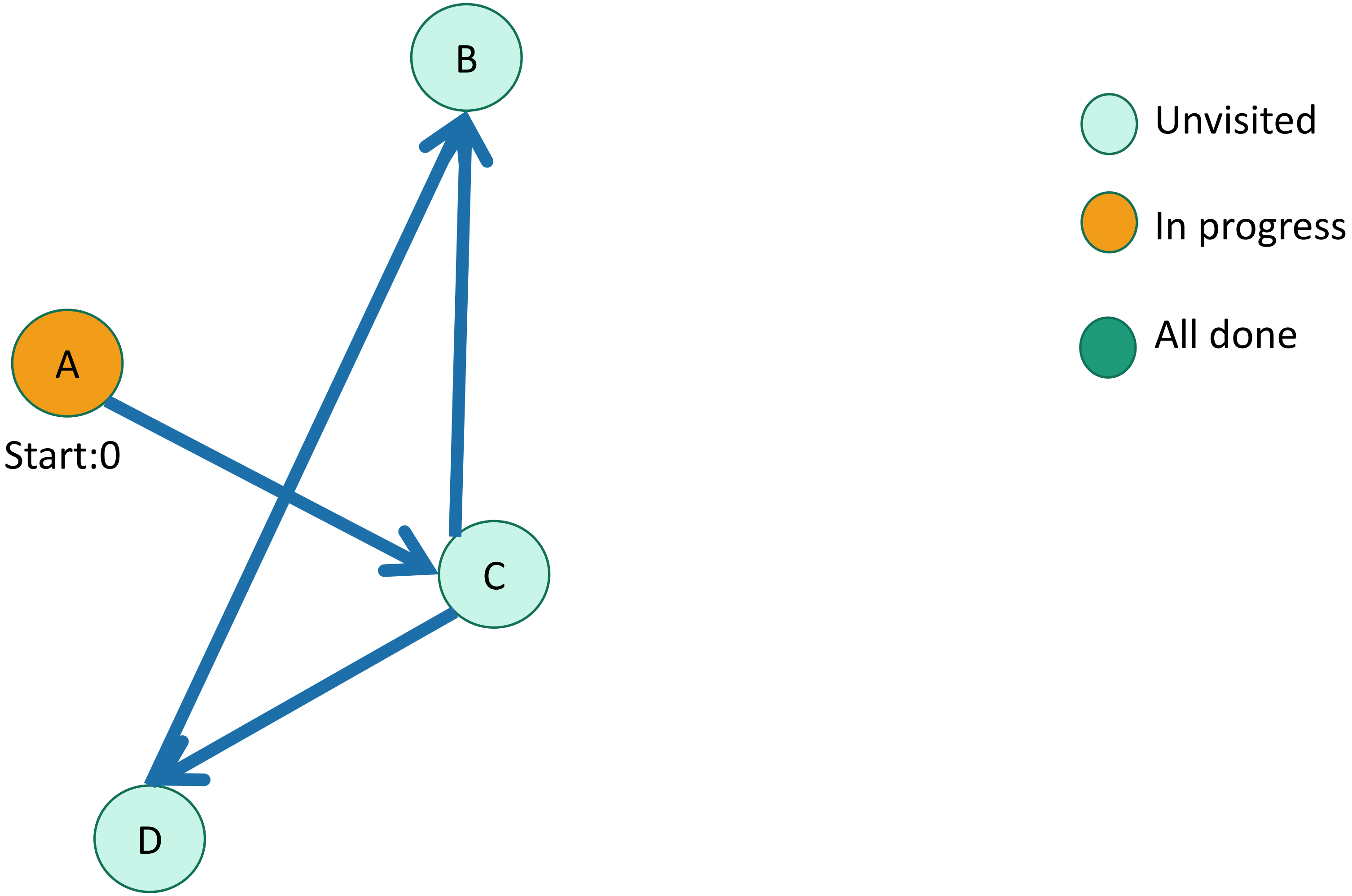
```
dkpg
tar
coreutils
libbz2
libselinux1
multiarch_support
```

What did we just learn?

- DFS can help you solve the **topological sorting problem**
  - That’s the fancy name for the problem of finding an ordering that respects all the dependencies
- Thinking about the DFS tree is helpful.

Example:

This example skipped in class – here for reference.



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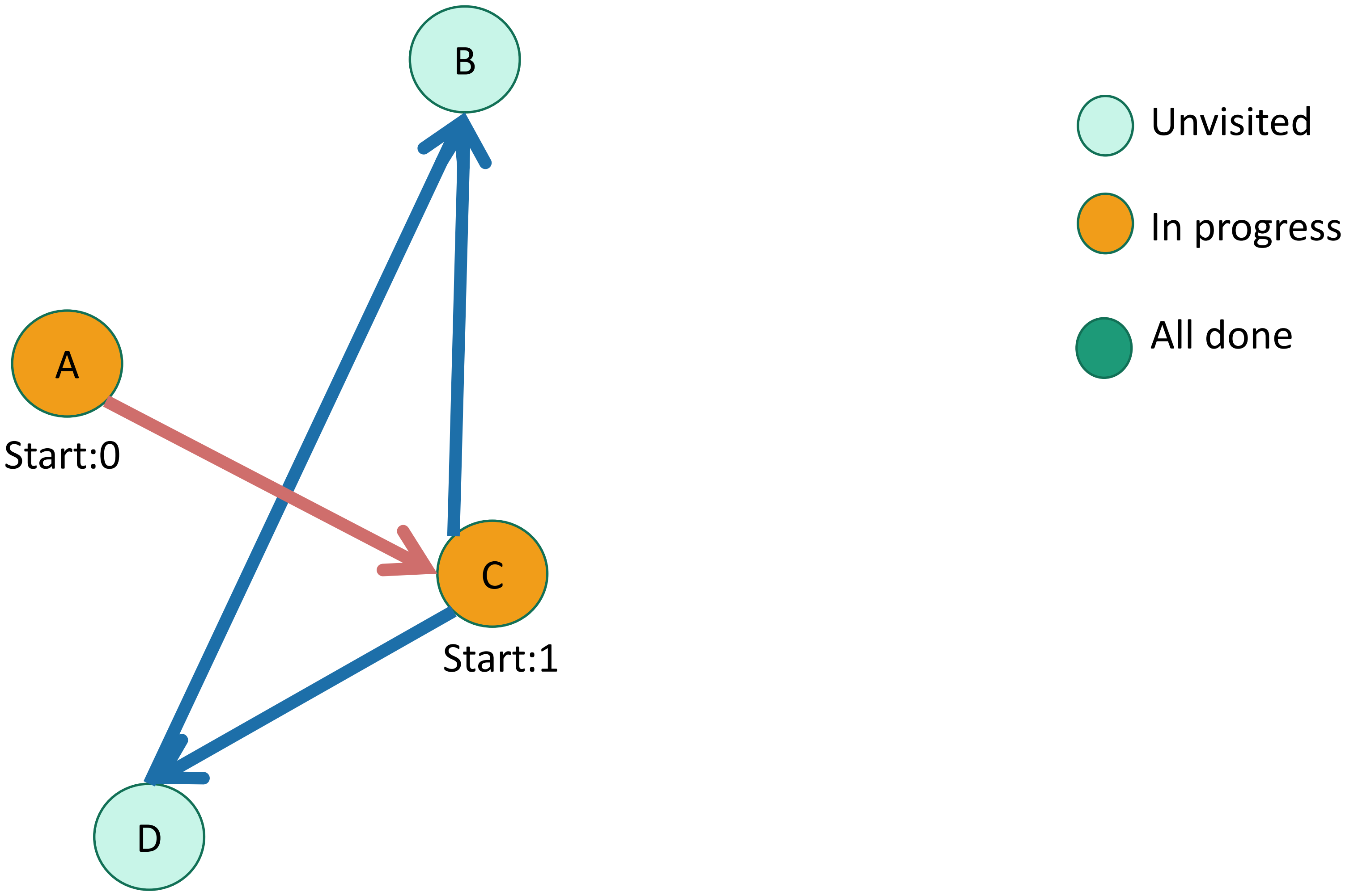

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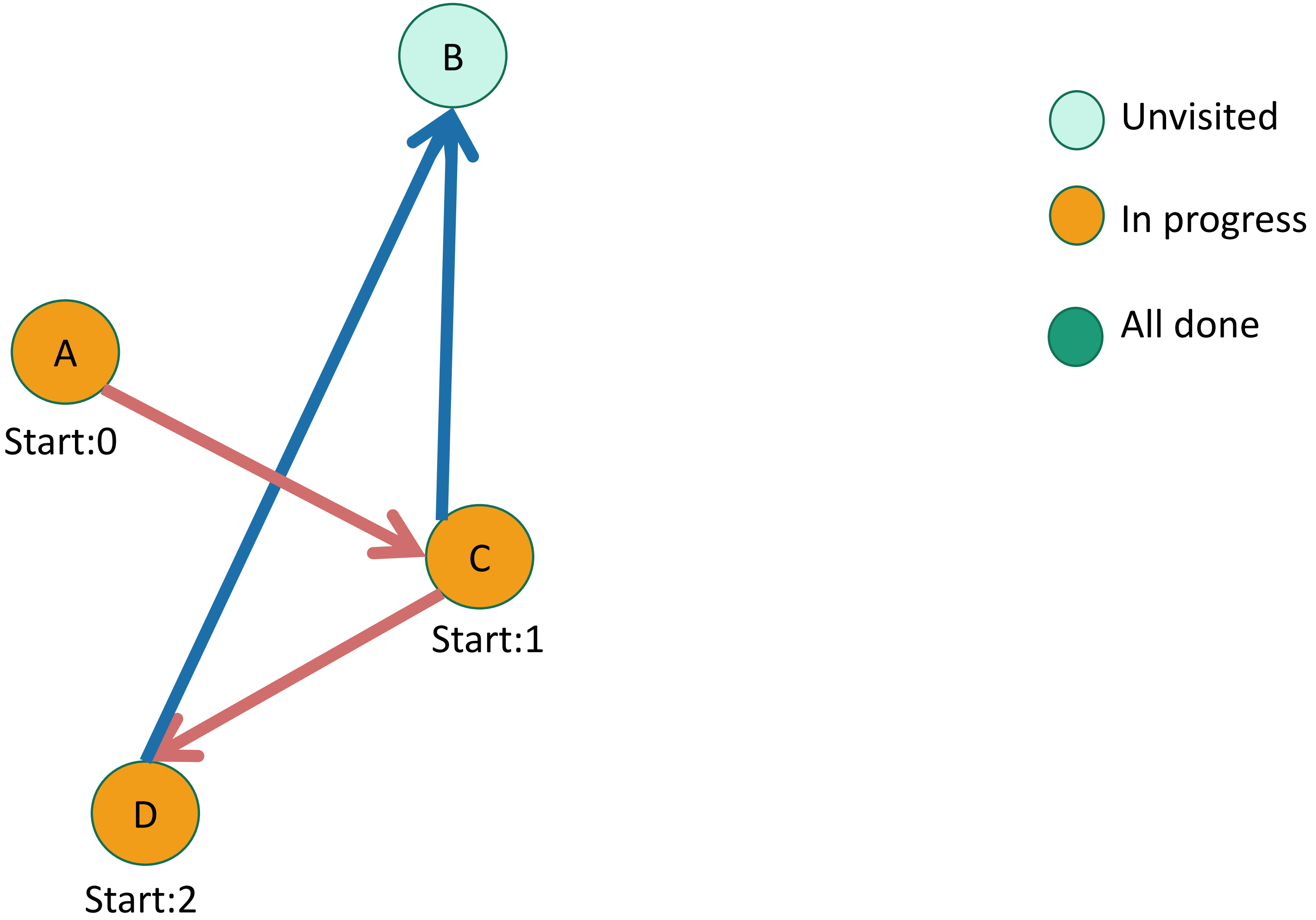

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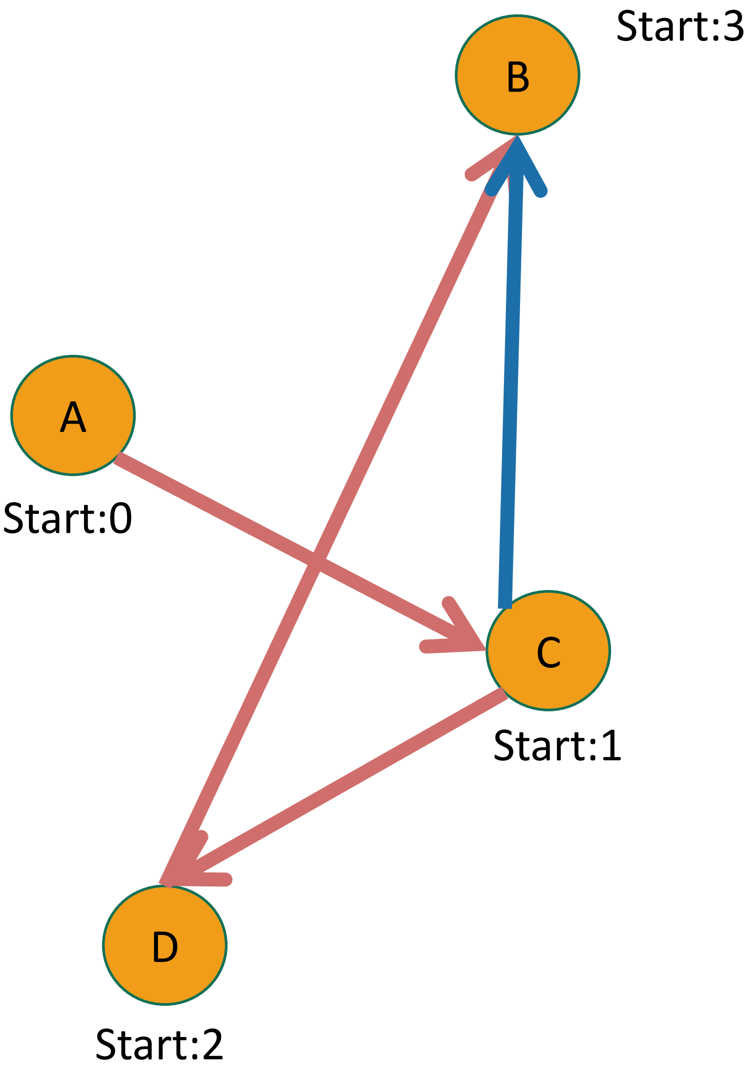

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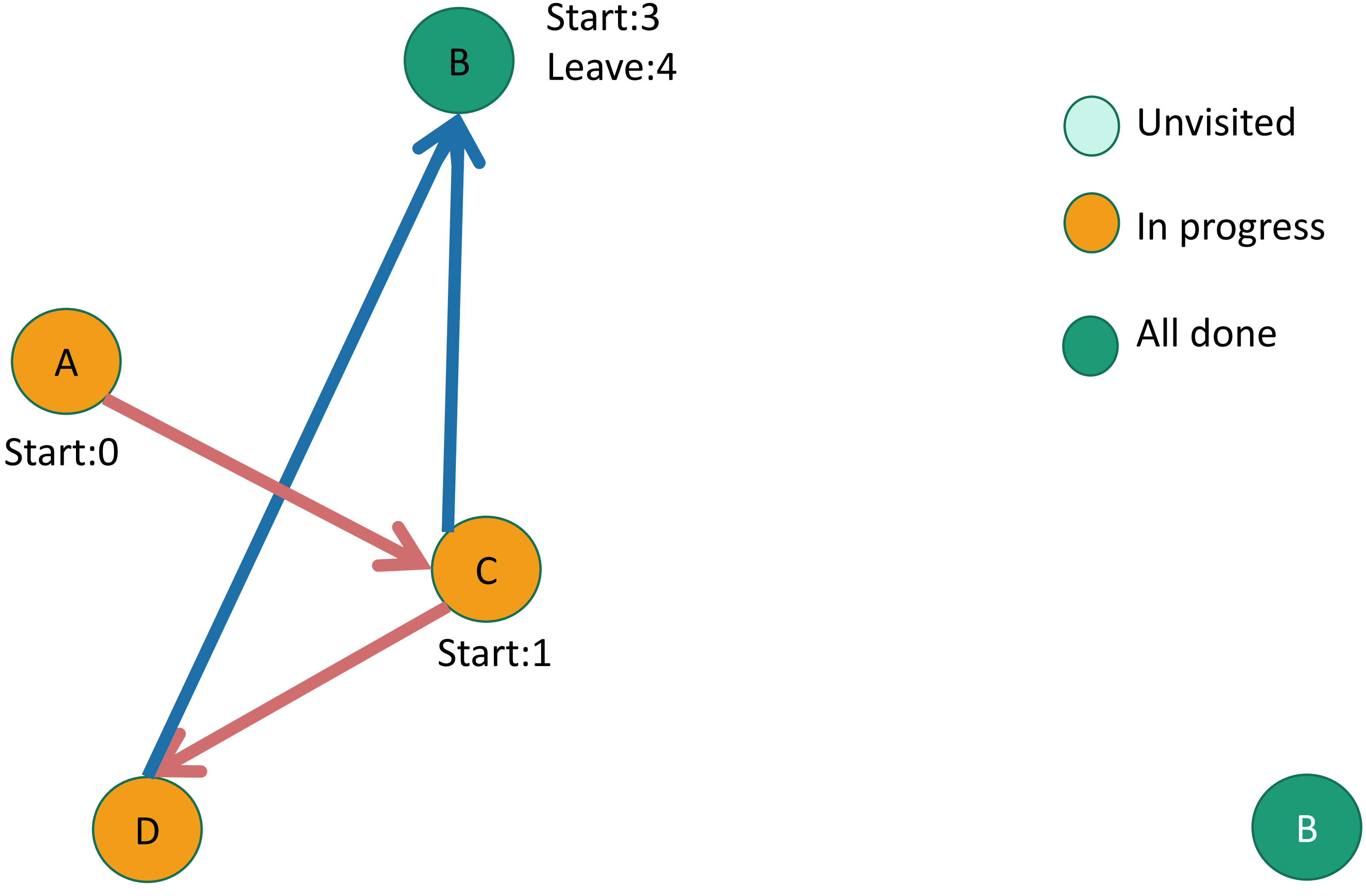

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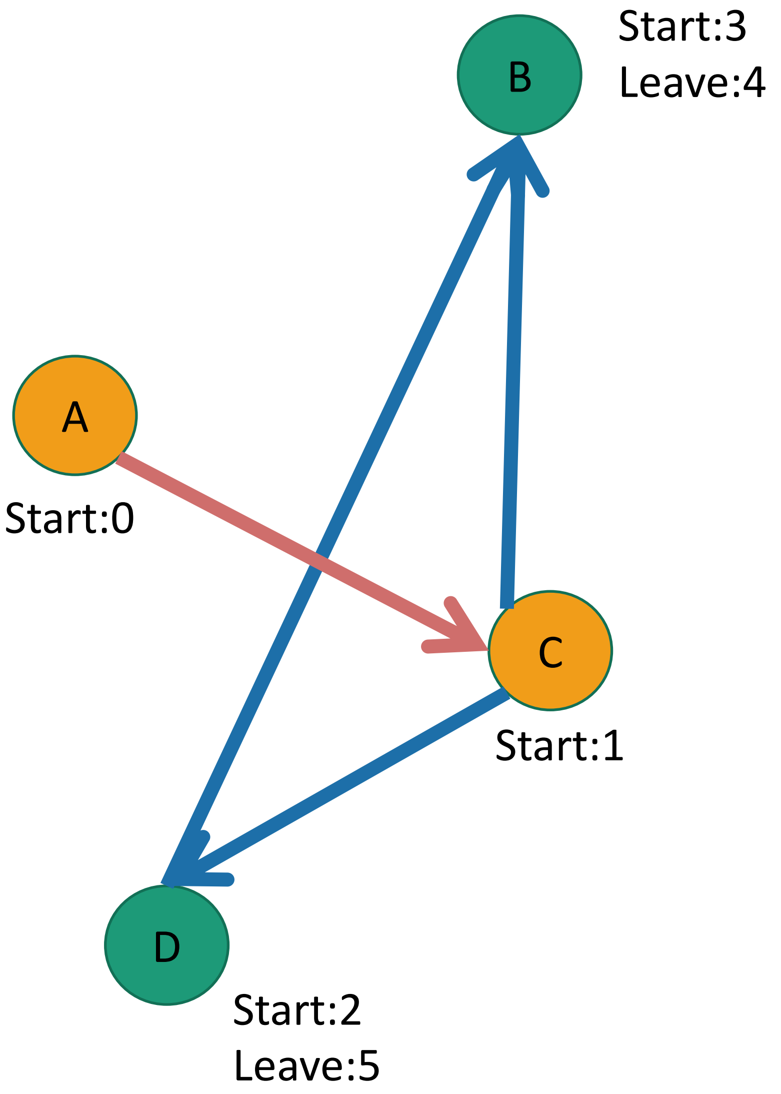

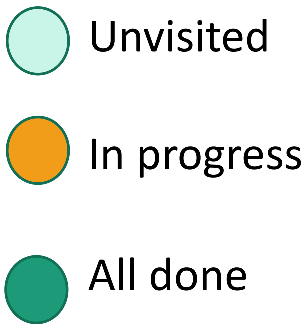

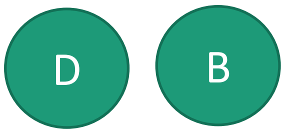

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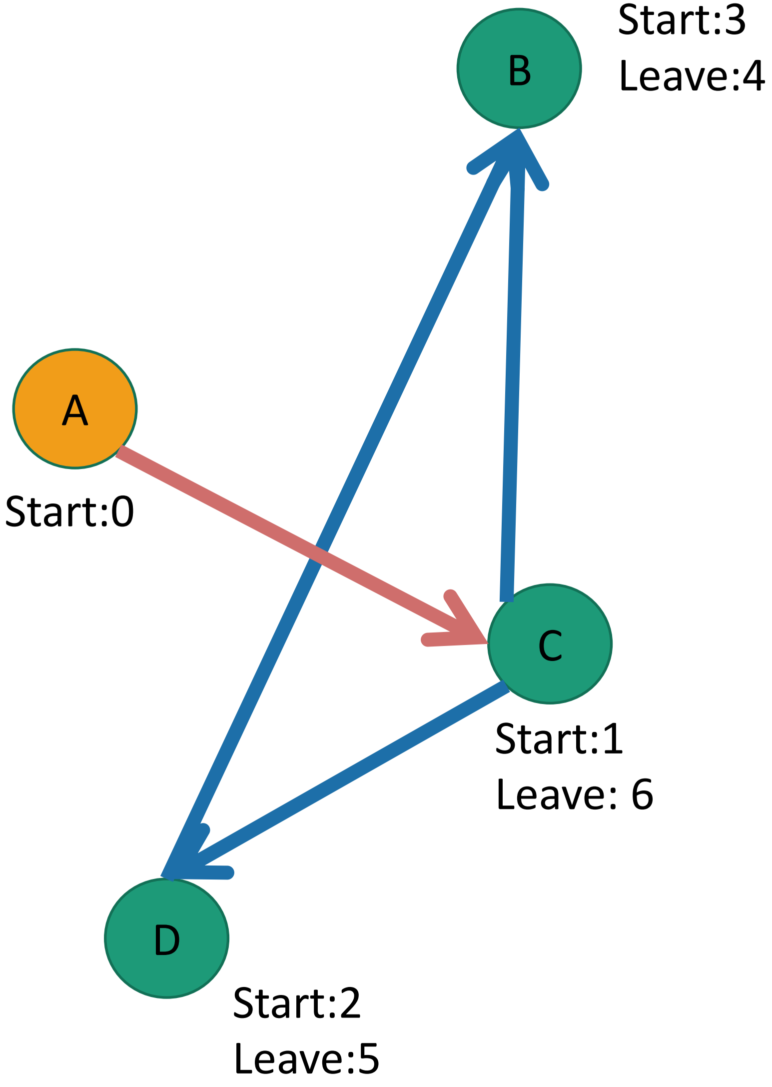

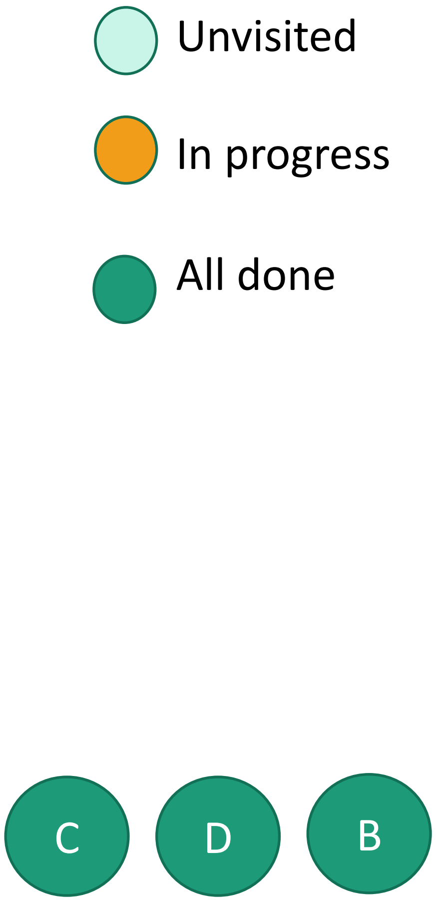

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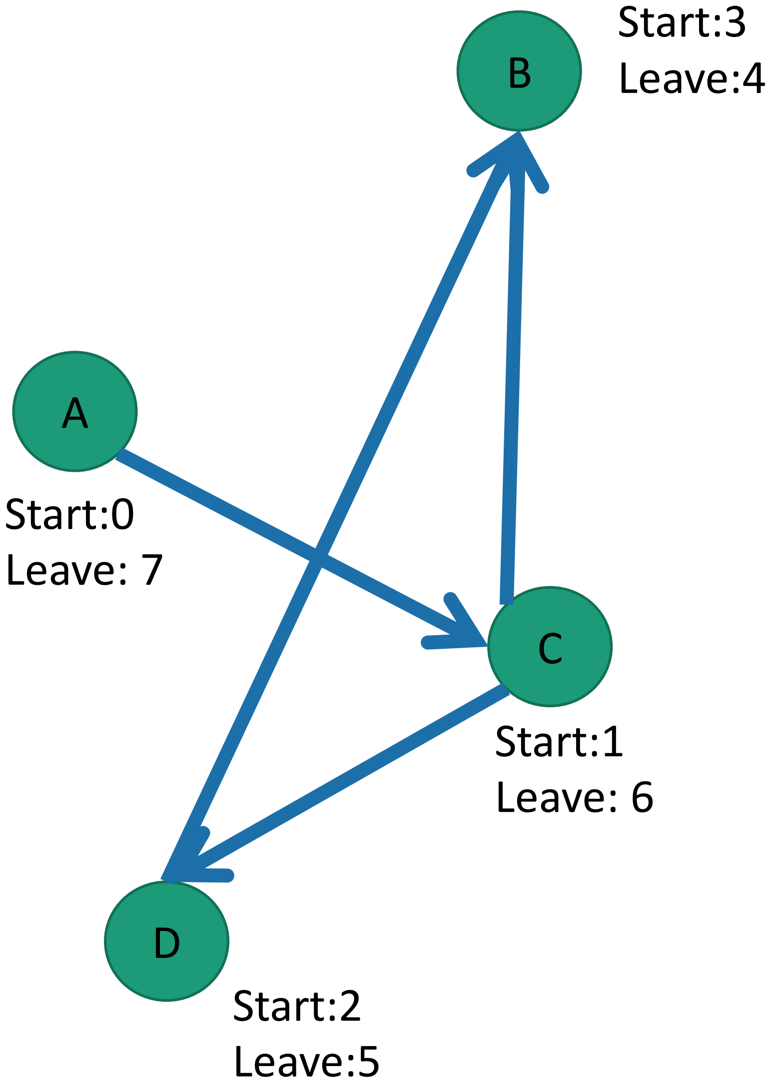
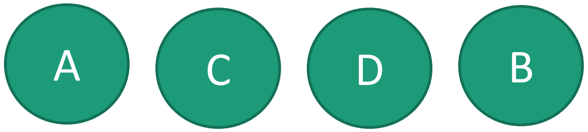


○ Unvisited

● In progress

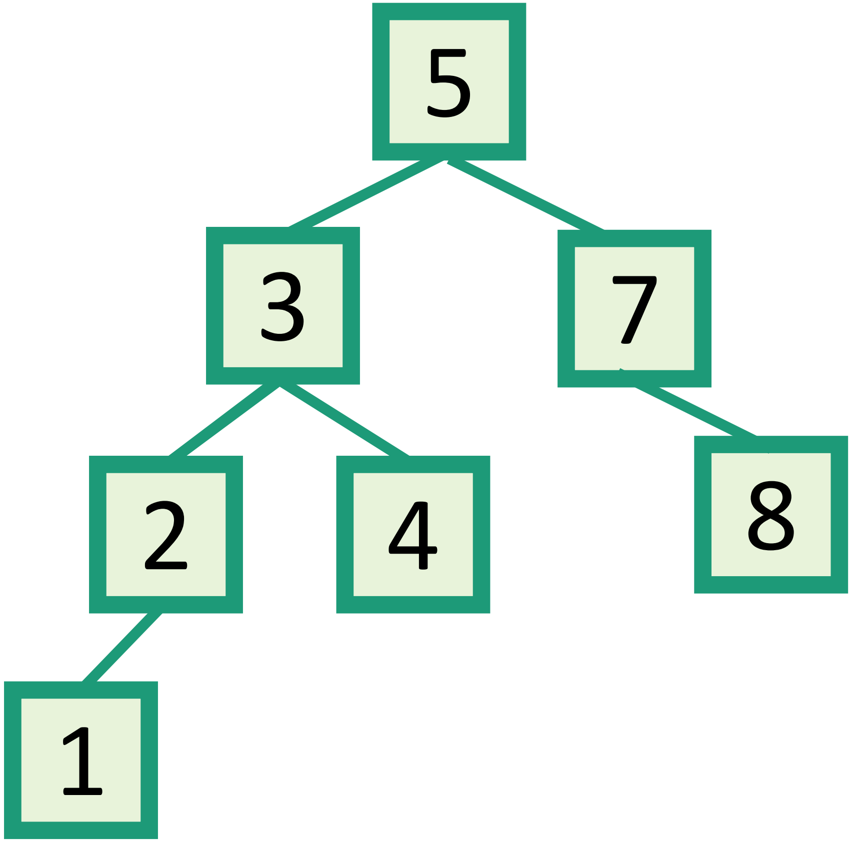
● All done

Do them in this order:



Another use of DFS that we’ve already seen

- In-order enumeration of binary search trees

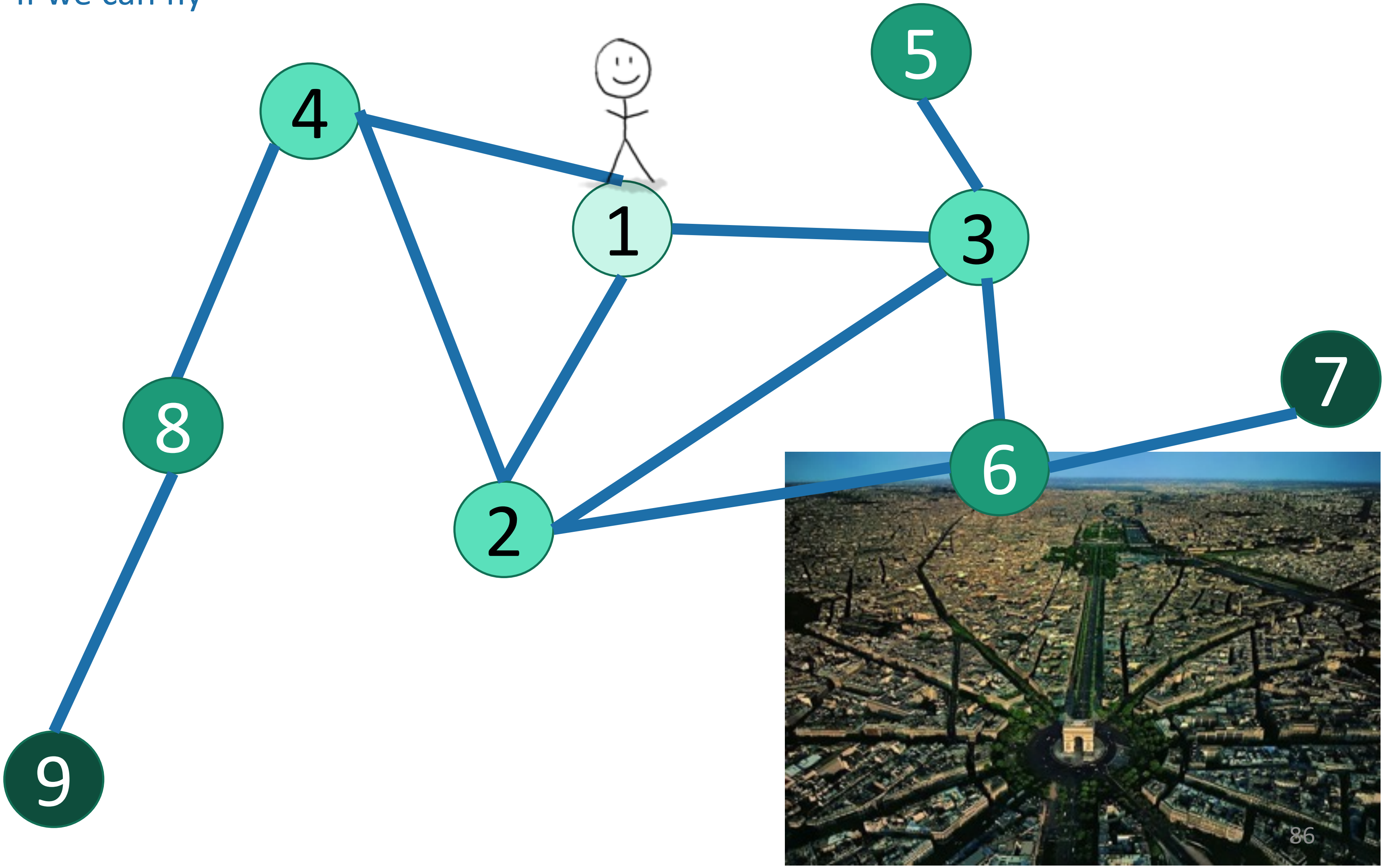


Do DFS and print a node’s label when you are done with the left child and before you begin the right child.

Part 2: breadth-first search

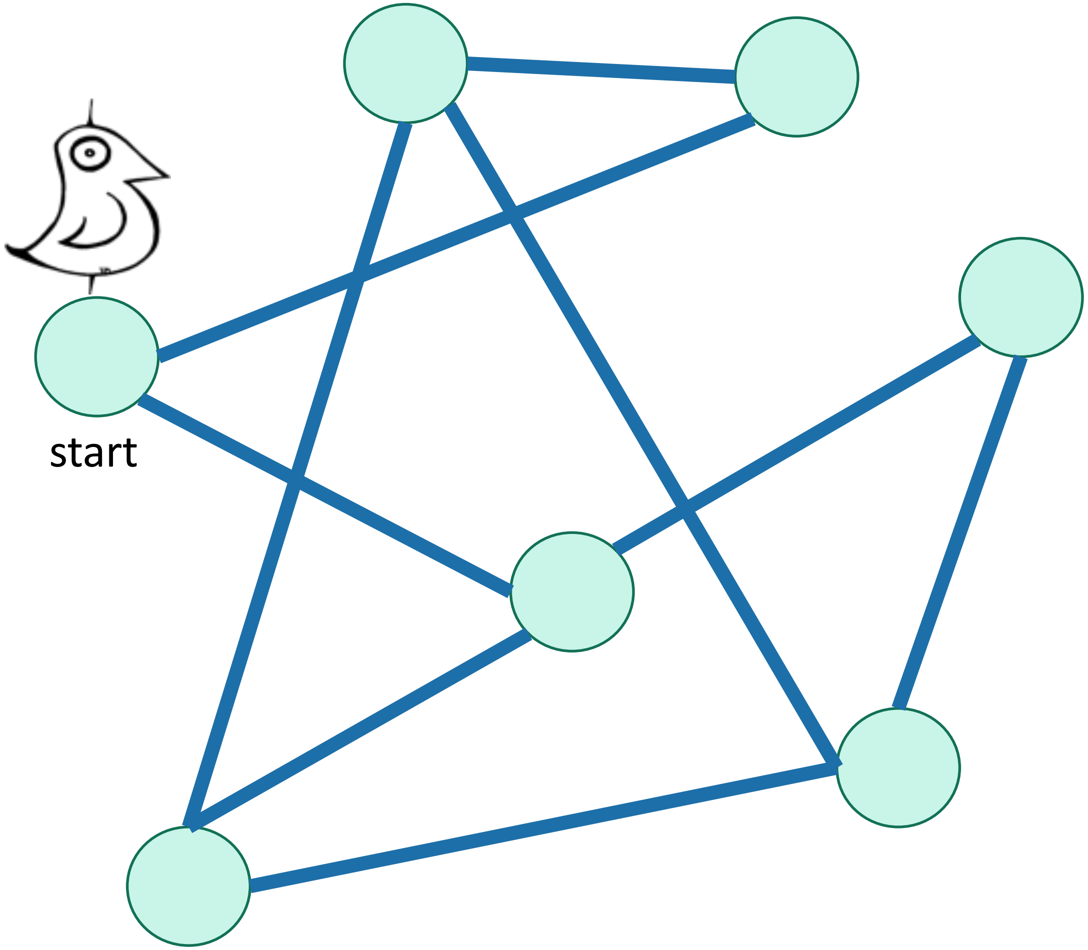
How do we explore a graph?

If we can fly



Breadth-First Search

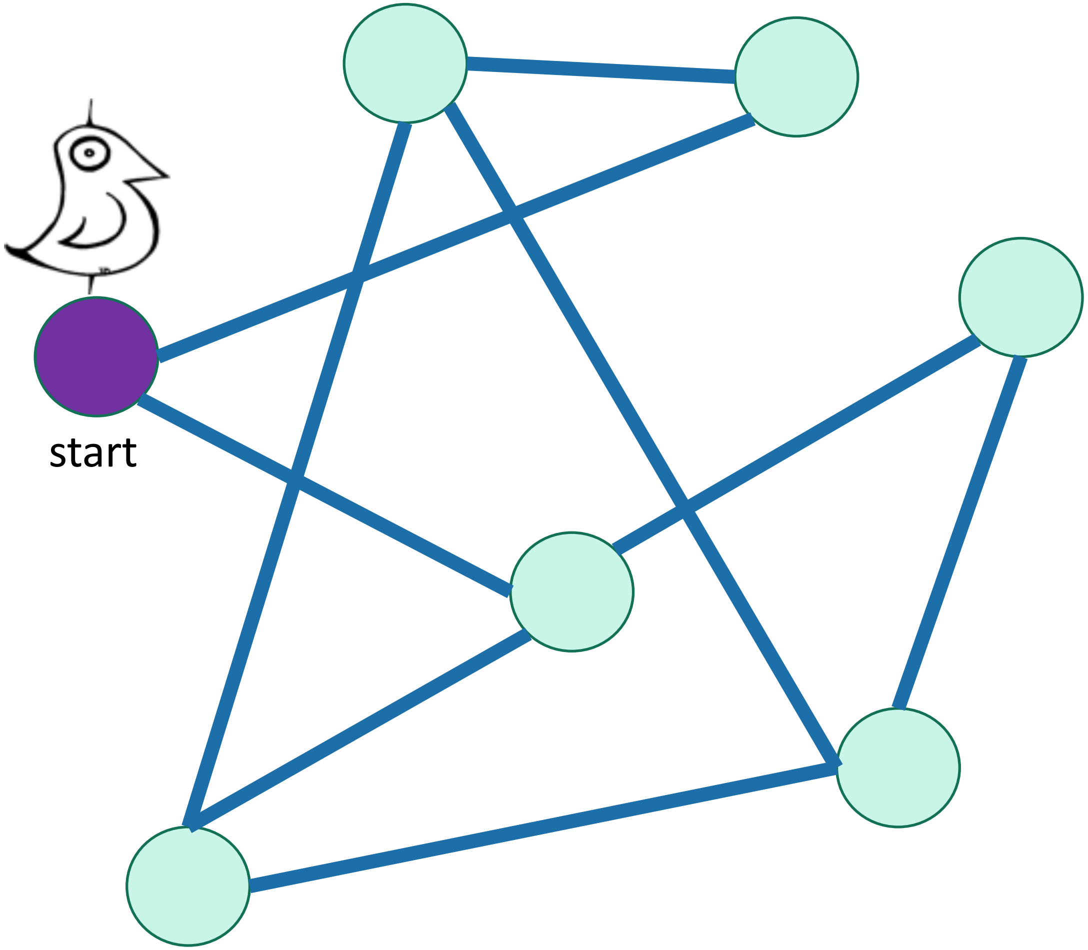
Exploring the world with a bird's-eye view



- ○ Not been there yet
- 🟣 Can reach there in zero steps
- 🔴 Can reach there in one step
- 🟠 Can reach there in two steps
- 🟢 Can reach there in three steps

Breadth-First Search

Exploring the world with a bird’s-eye view



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Breadth-First Search

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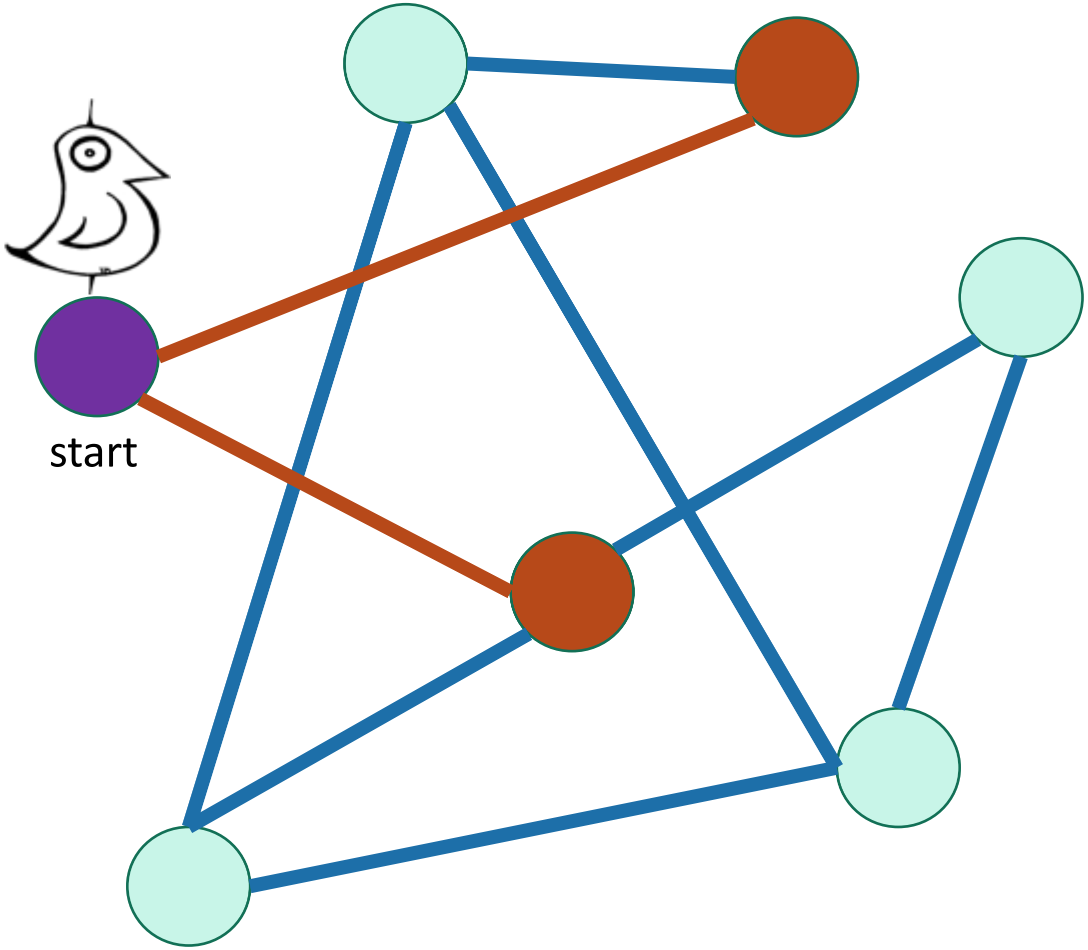

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Breadth-First Search

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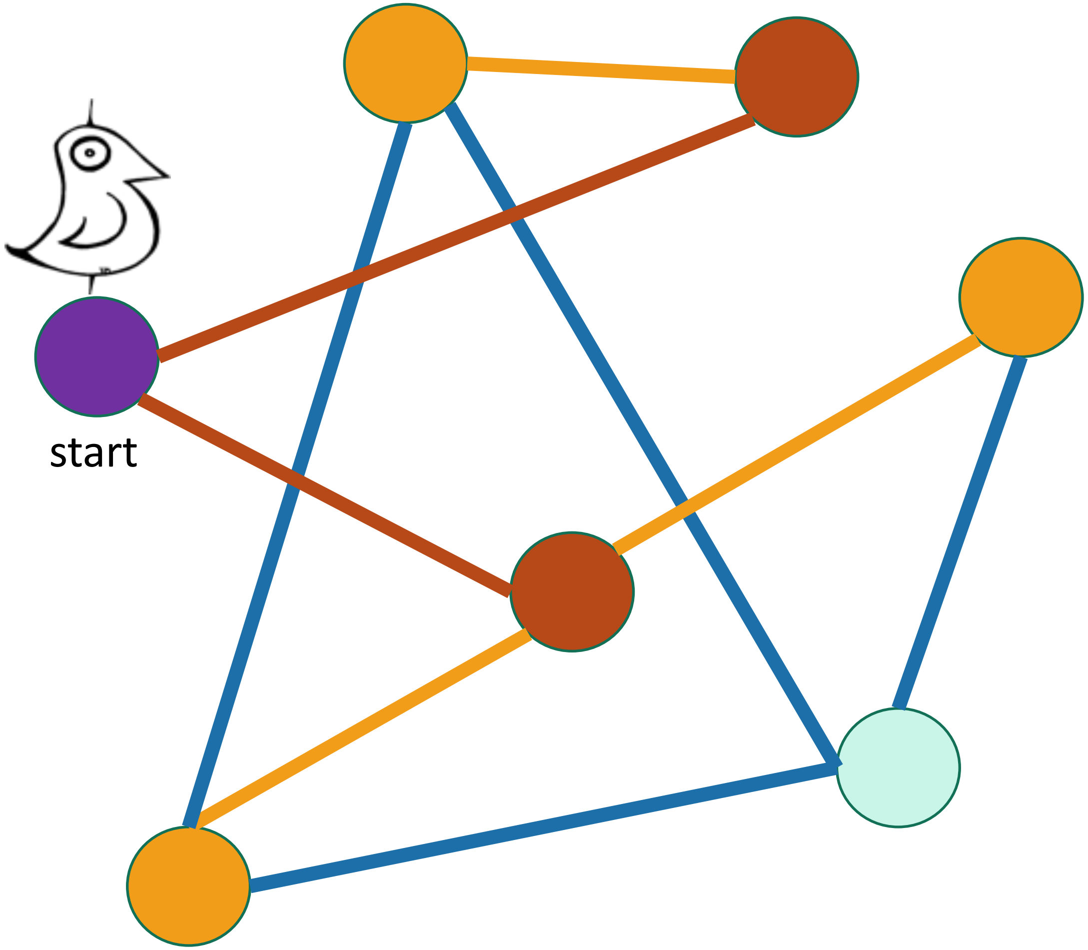

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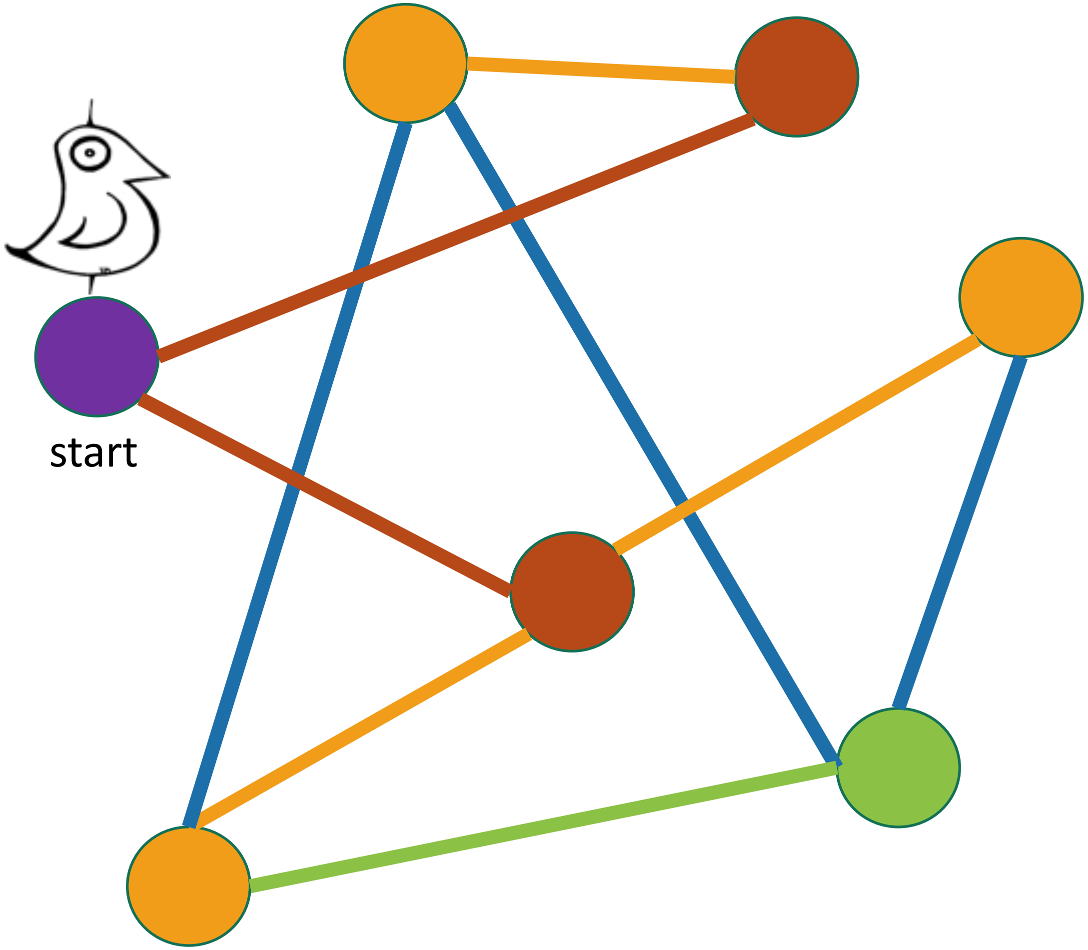
Breadth-First Search

Exploring the world with a bird’s-eye view



○ Not been there yet

🟣 Can reach there in zero steps

🟤 Can reach there in one step

🟠 Can reach there in two steps

🟢 Can reach there in three steps

World:

EXPLORED!

Same disclaimer as for DFS: you may have seen other ways to implement this, this will be convenient for us.

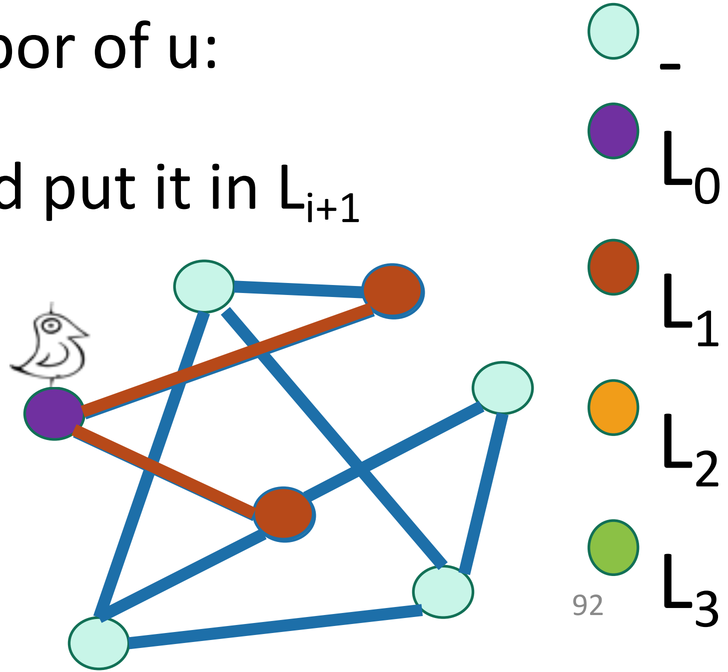
Breadth-First Search

Exploring the world with pseudocode

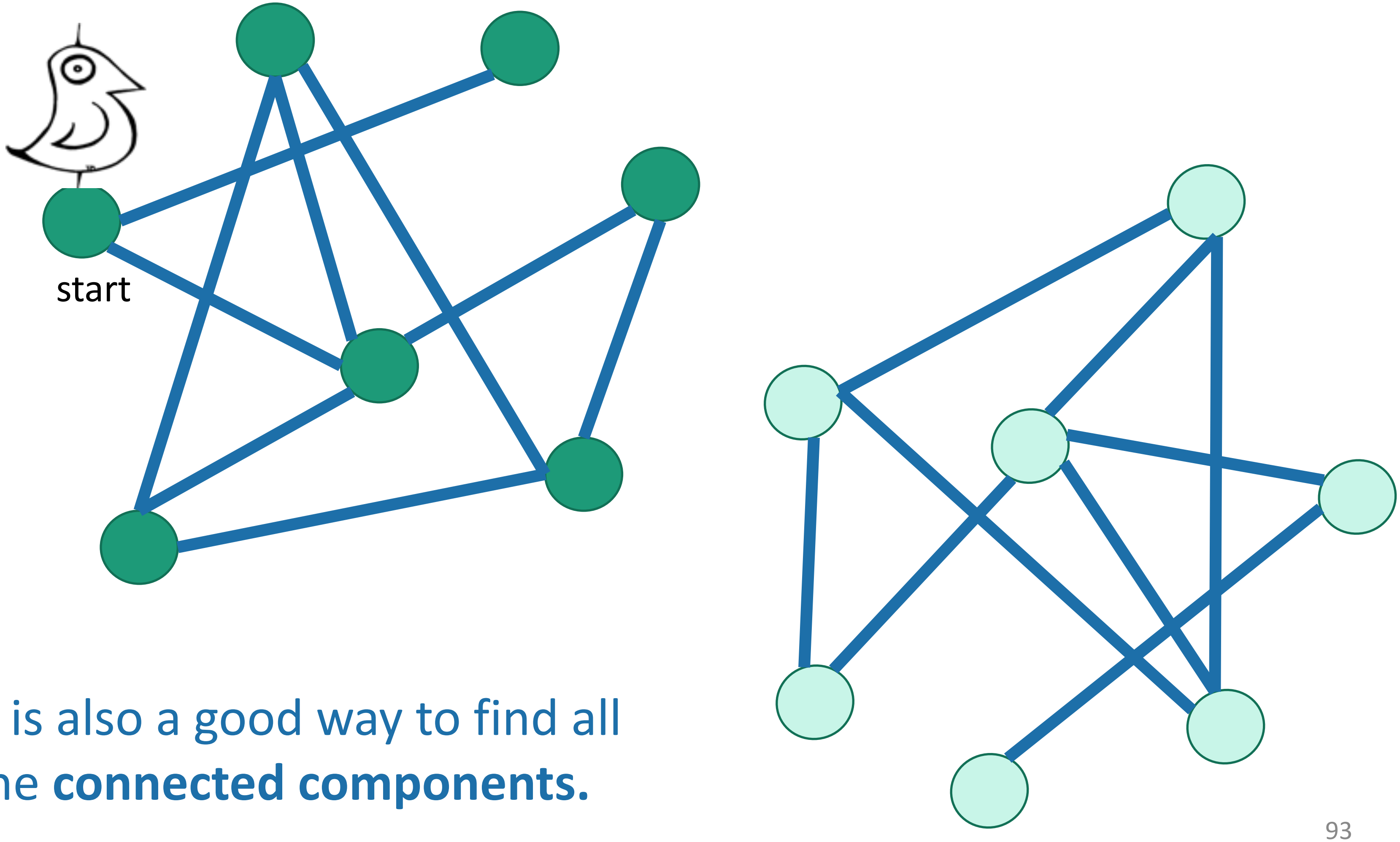
L_i is the set of nodes we can reach in i steps from w

```
• Set $L_i$ = [] for i=1,…,n
• $L_0$ = [w], where w is the start node
• Mark w as visited
• For i = 0, …, n-1:
    • For u in $L_i$:
        • For each v which is a neighbor of u:
            • If v isn't yet visited:
                • mark v as visited, and put it in $L_{i+1}$
```

Go through all the nodes in L_i and add their unvisited neighbors to L_{i+1}



BFS also finds all the nodes reachable from the starting point



It is also a good way to find all the **connected components.**

Running time and extension to directed graphs

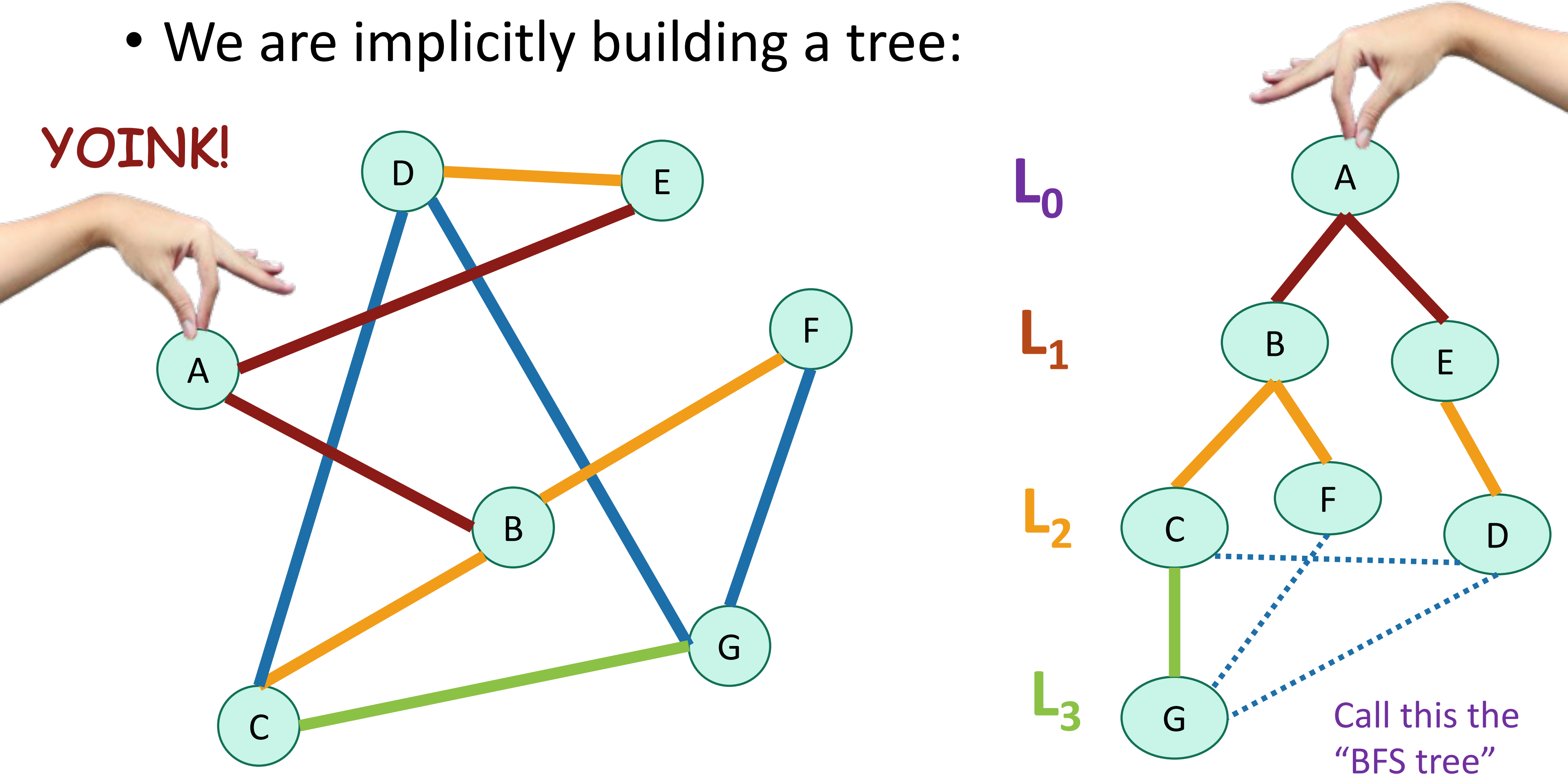
- To explore the whole graph, explore the connected components one-by-one.
  - Same argument as DFS: BFS running time is O(n + m)
- Like DFS, BFS also works fine on directed graphs.

Verify these!

Siggi the Studious Stork

Why is it called breadth-first?

- We are implicitly building a tree:

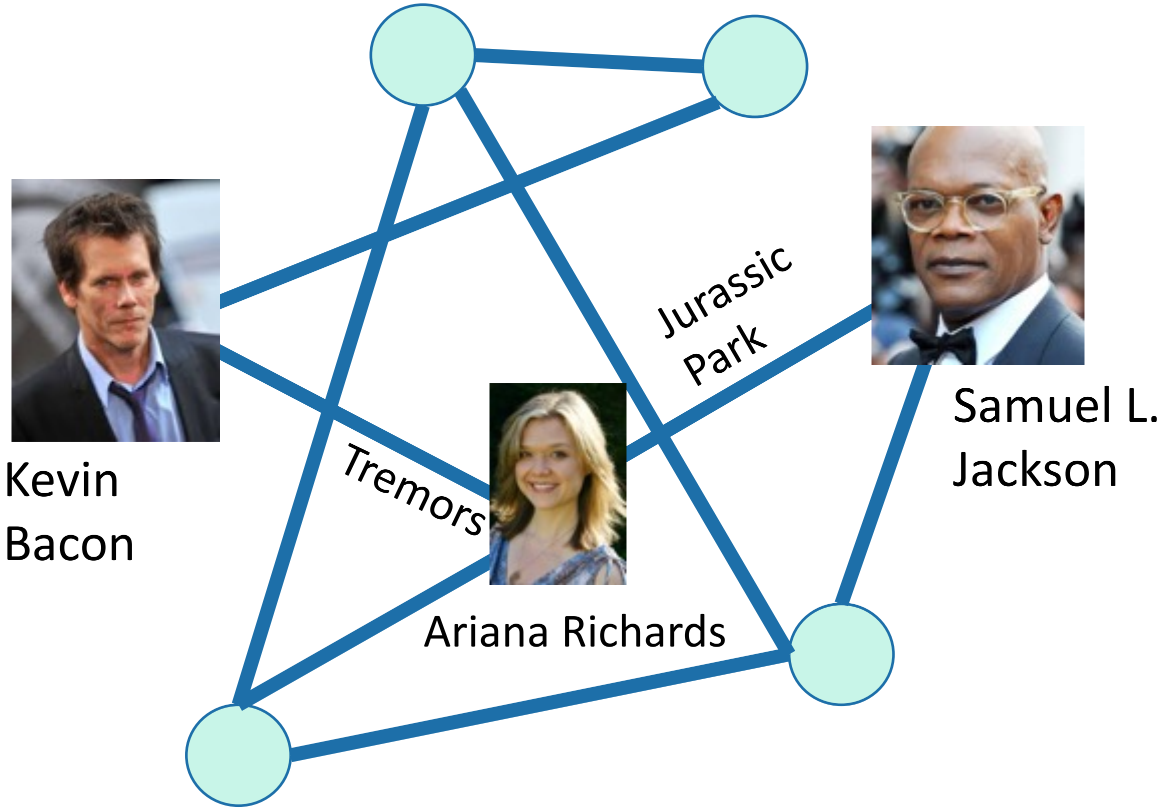


Call this the "BFS tree"

- First we go as broadly as we can.

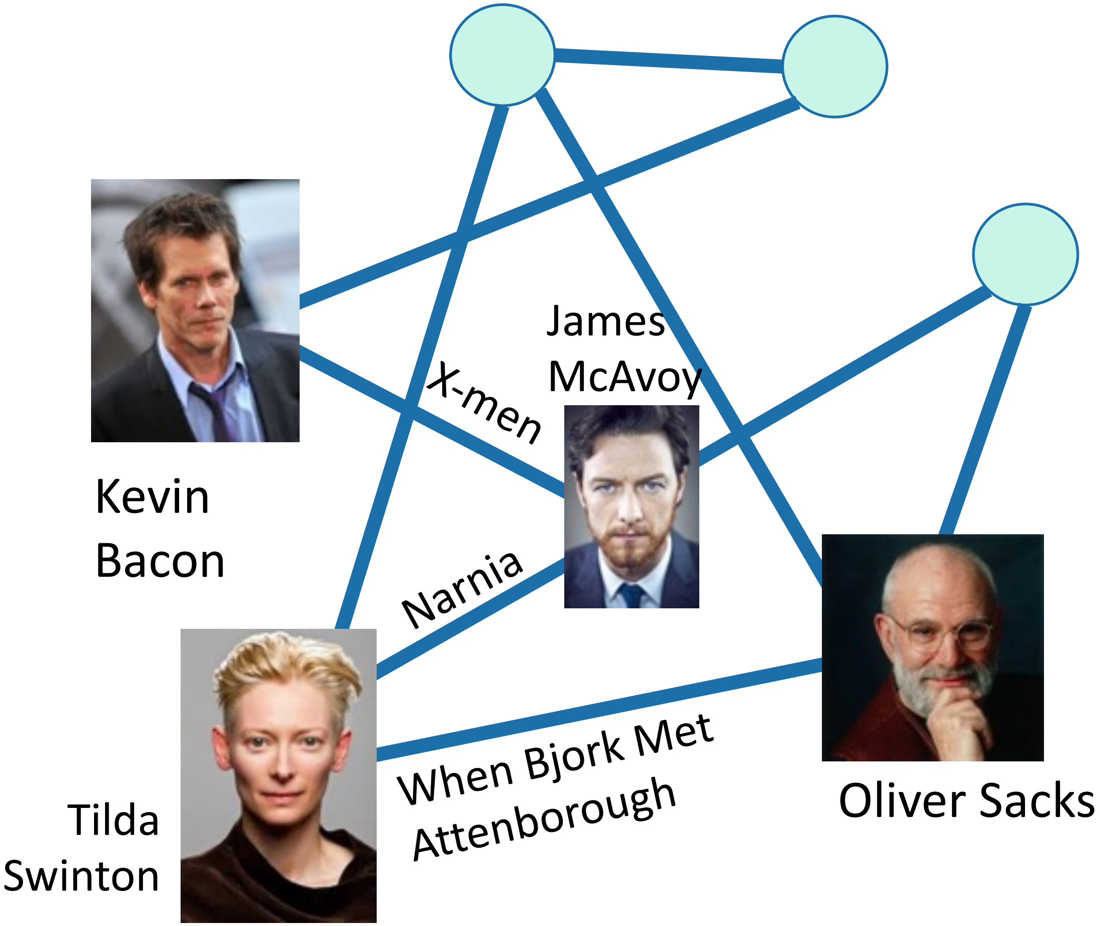
Pre-lecture exercise

- What Samuel L. Jackson’s Bacon number?



(Answer: 2)

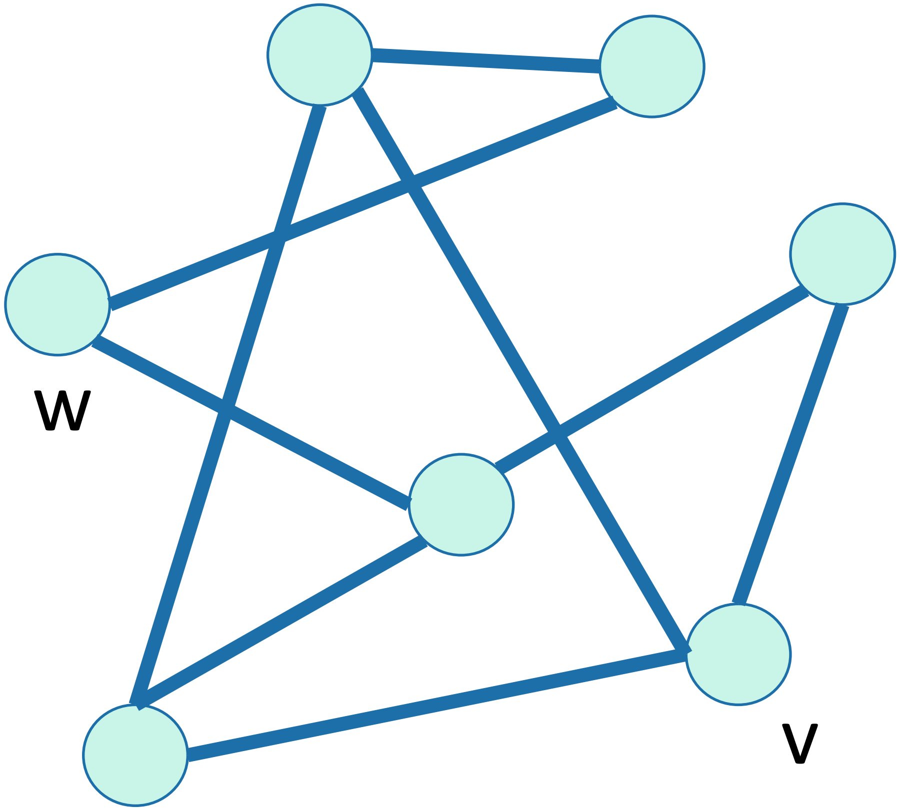
An example with distance 3



It is really hard to find people with Bacon number 3!

Application of BFS: shortest path

- How long is the shortest path between w and v?



Application of BFS: shortest path

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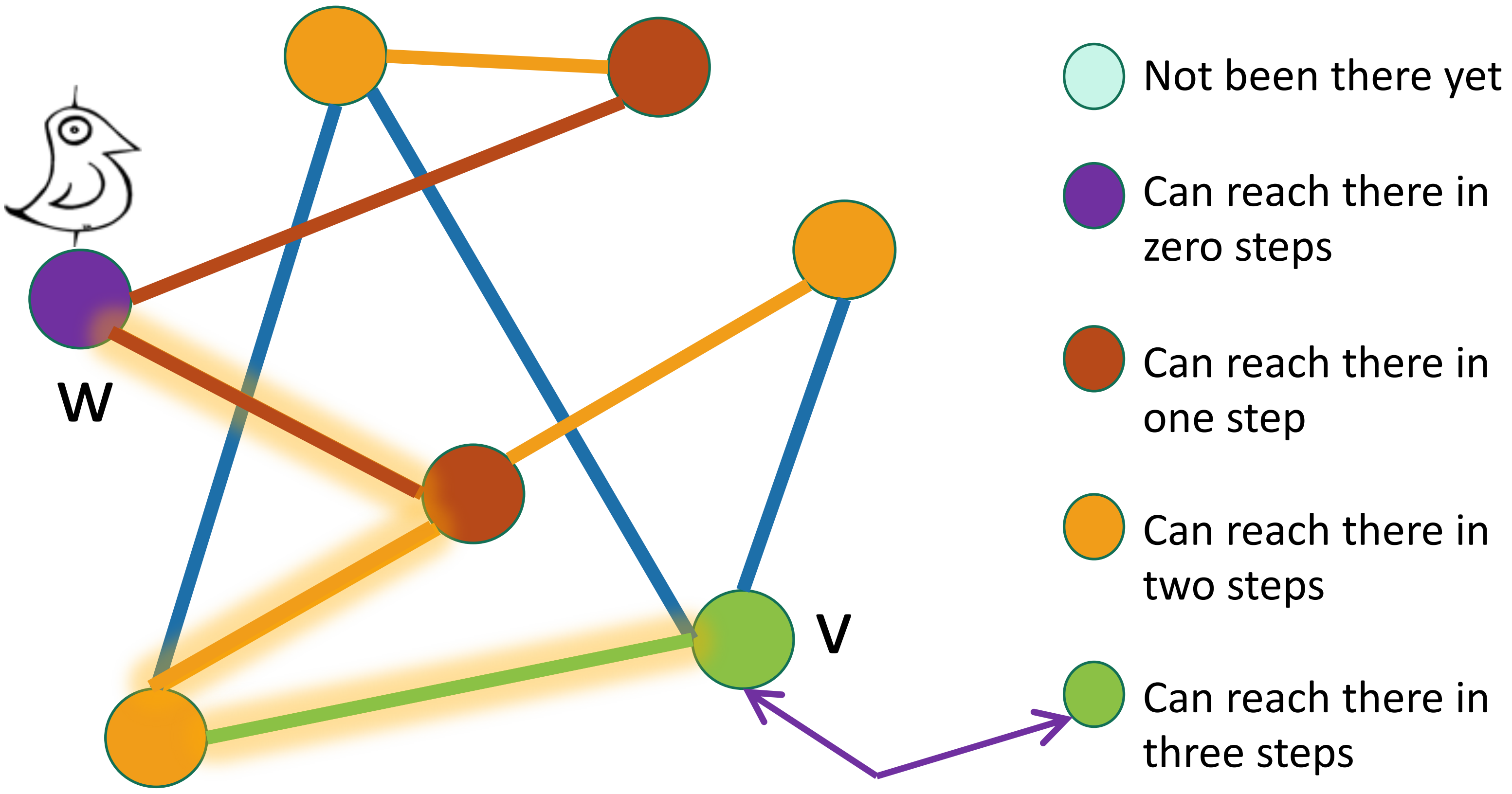


○ Not been there yet

● Can reach there in zero steps

● Can reach there in one step

● Can reach there in two steps

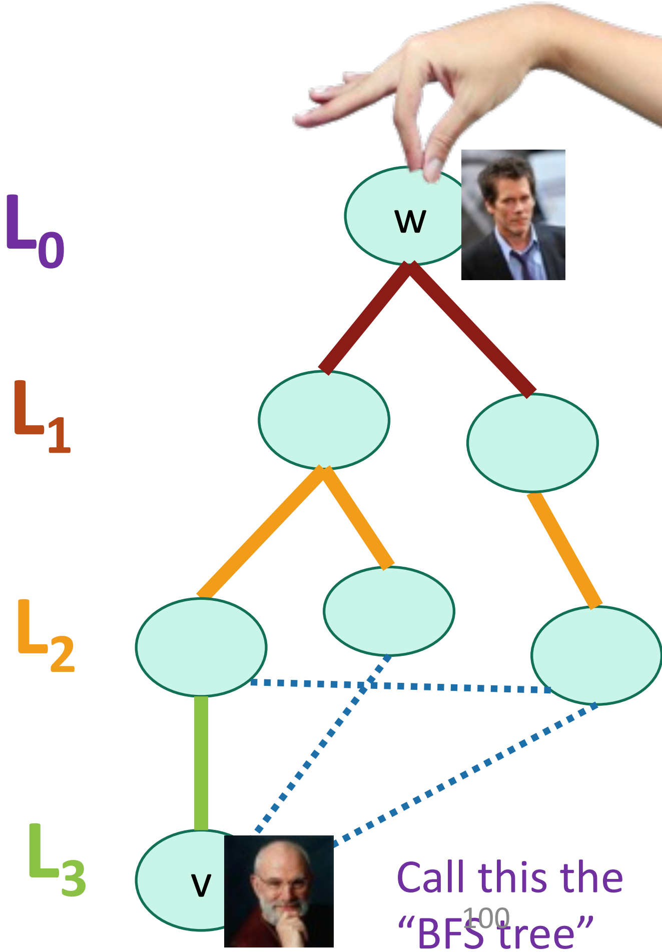
● Can reach there in three steps

It's three!

To find the distance between w and all other vertices v

The **distance** between two vertices is the number of edges in the shortest path between them.

- Do a BFS starting at w
- For all v in L_i
  - The shortest path between w and v has length i.
  - A shortest path between w and v is given by the path in the BFS tree.
- If we never found v, the distance is infinite.



Call this the "BFS tree"

Modify the BFS pseudocode to return shortest paths!
Prove that this indeed returns shortest paths!

Gauss has no Bacon number

What have we learned?

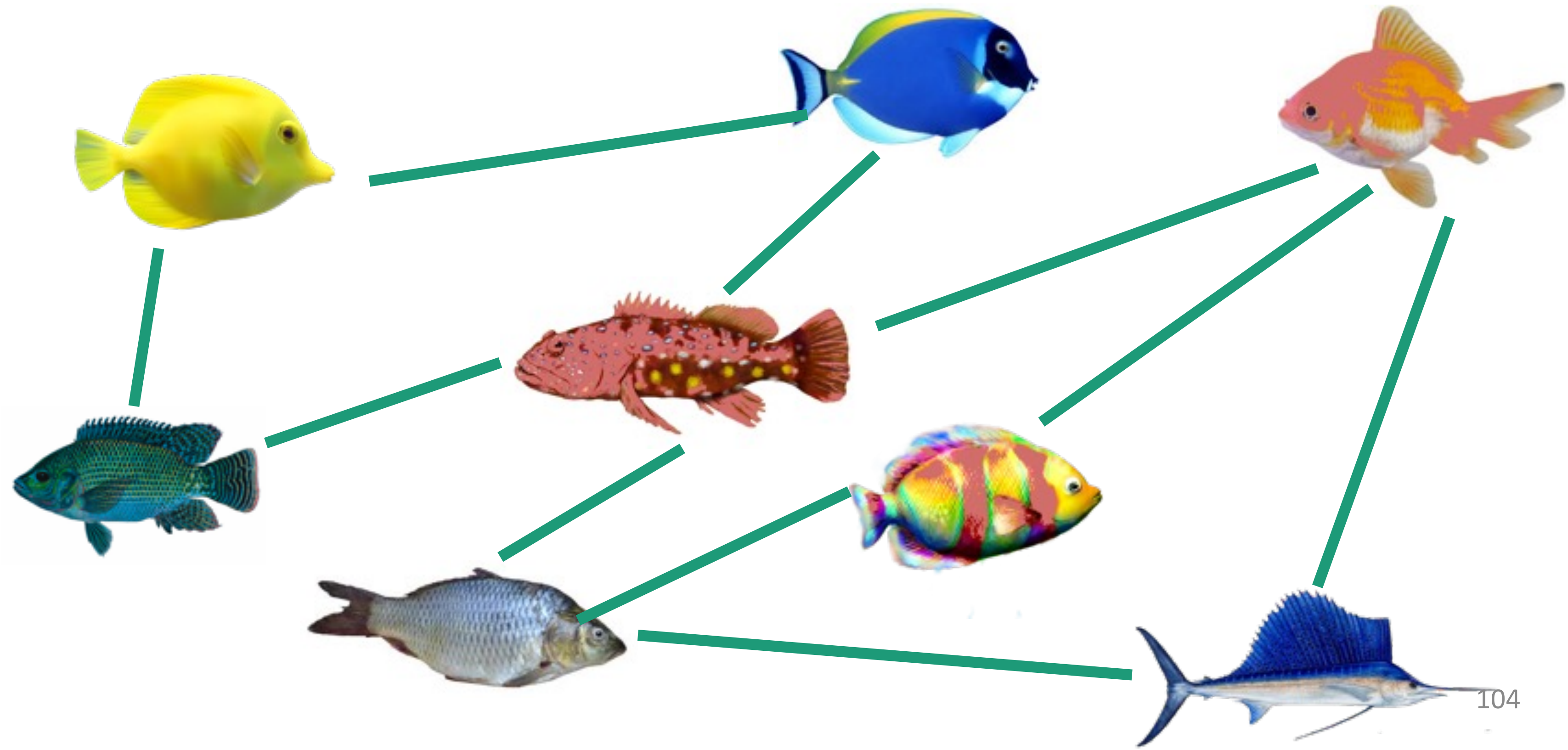
- The BFS tree is useful for computing distances between pairs of vertices.
- We can find the shortest path between u and v in time O(m).

Another application of BFS (if time)

- Testing bipartite-ness

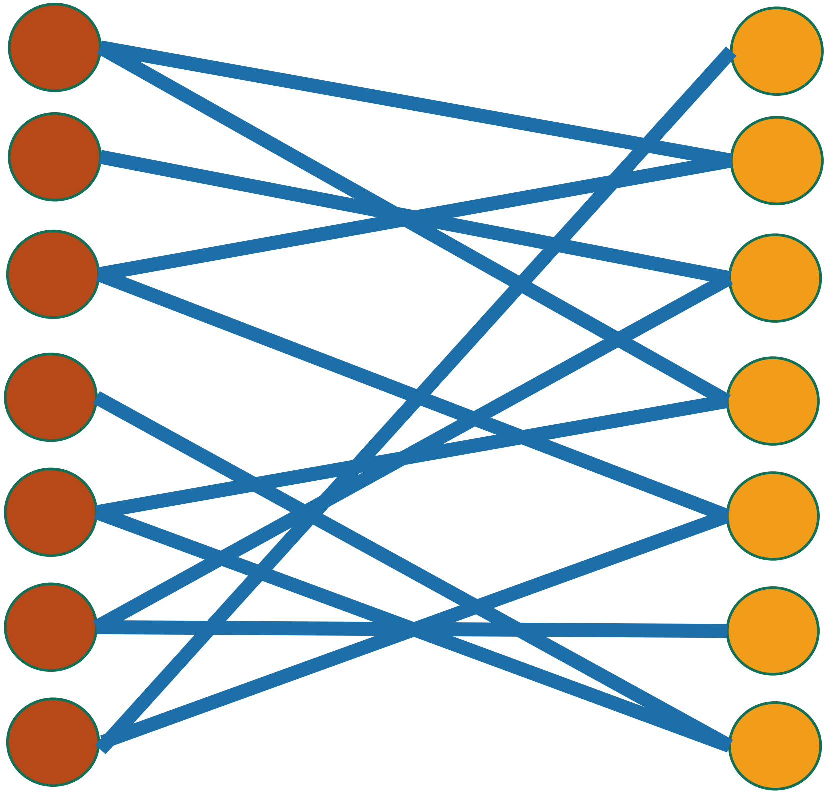
Pre-lecture exercise: fish

- You have a bunch of fish and two fish tanks.
- Some pairs of fish will fight if put in the same tank.
  - Model this as a graph: connected fish will fight.
- Can you put the fish in the two tanks so that there is no fighting?



Bipartite graphs

- A bipartite graph looks like this:



Can color the vertices red and orange so that there are no edges between any same-colored vertices

Example:

🔴 are in tank A

🟠 are in tank B

🔴–🟠 if the fish fight

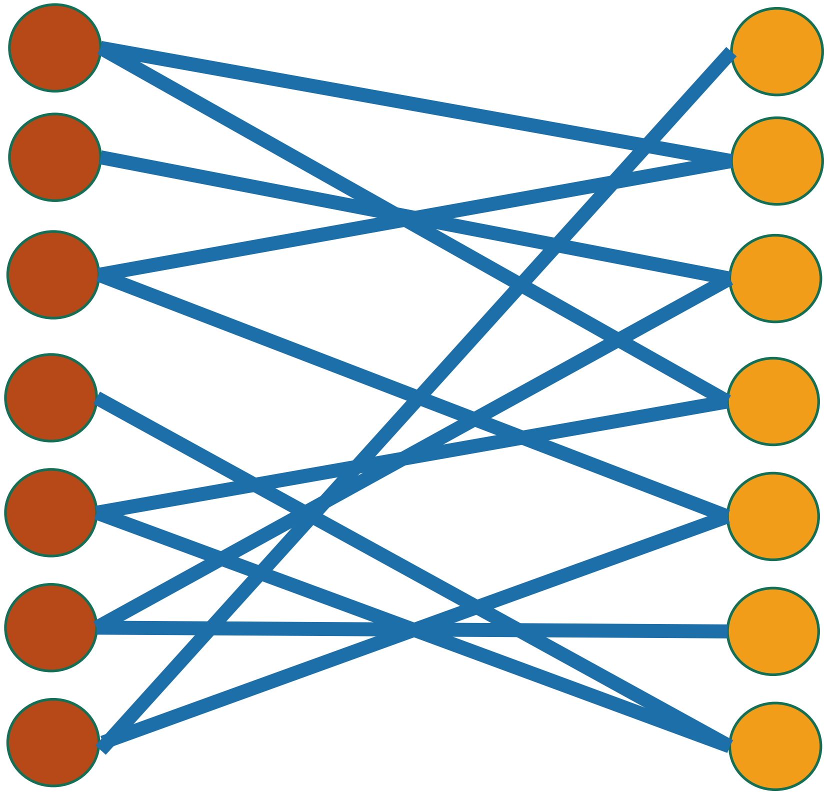
Example:

🔴 are students

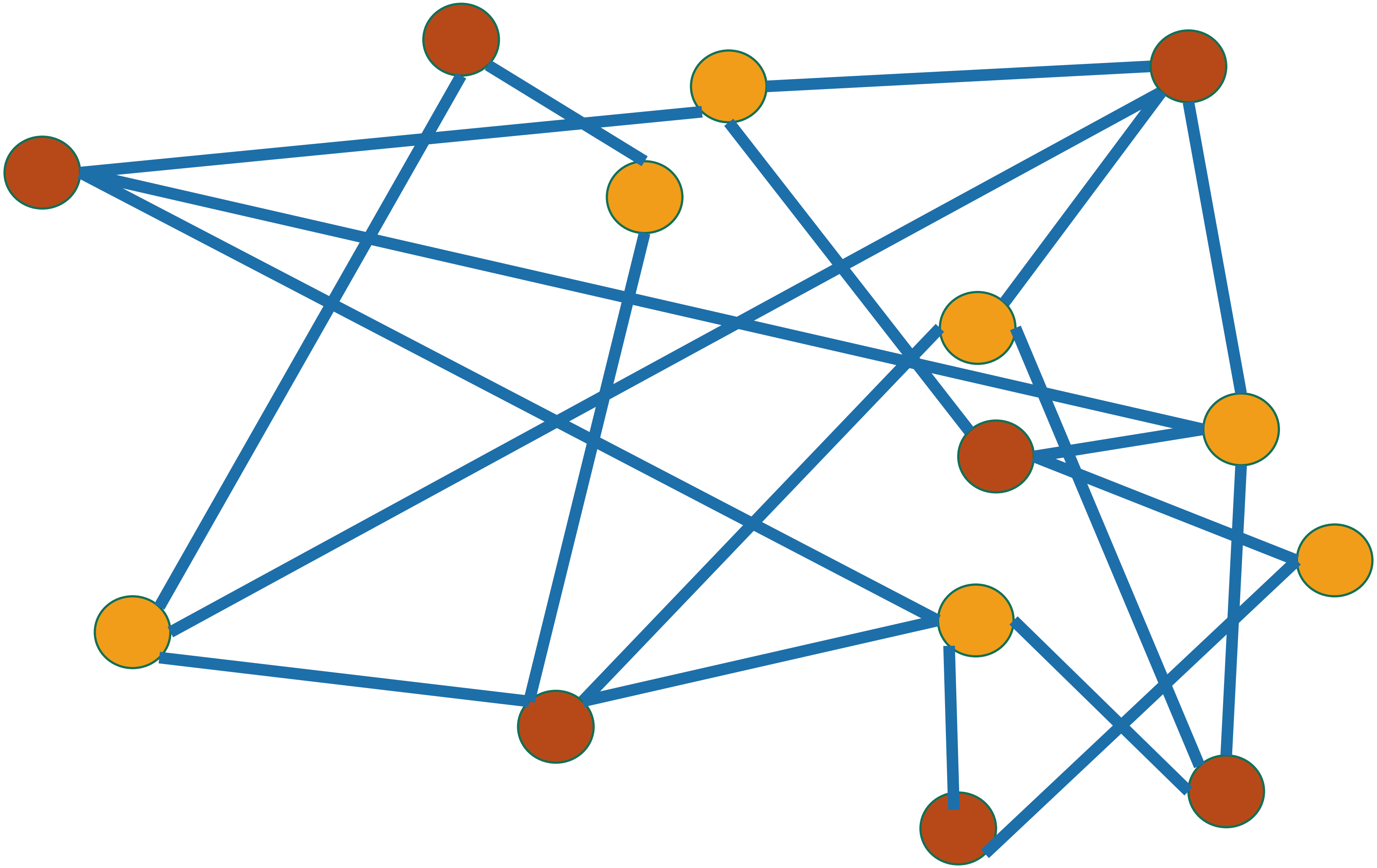
🟠 are classes

🔴–🟠 if the student is enrolled in the class

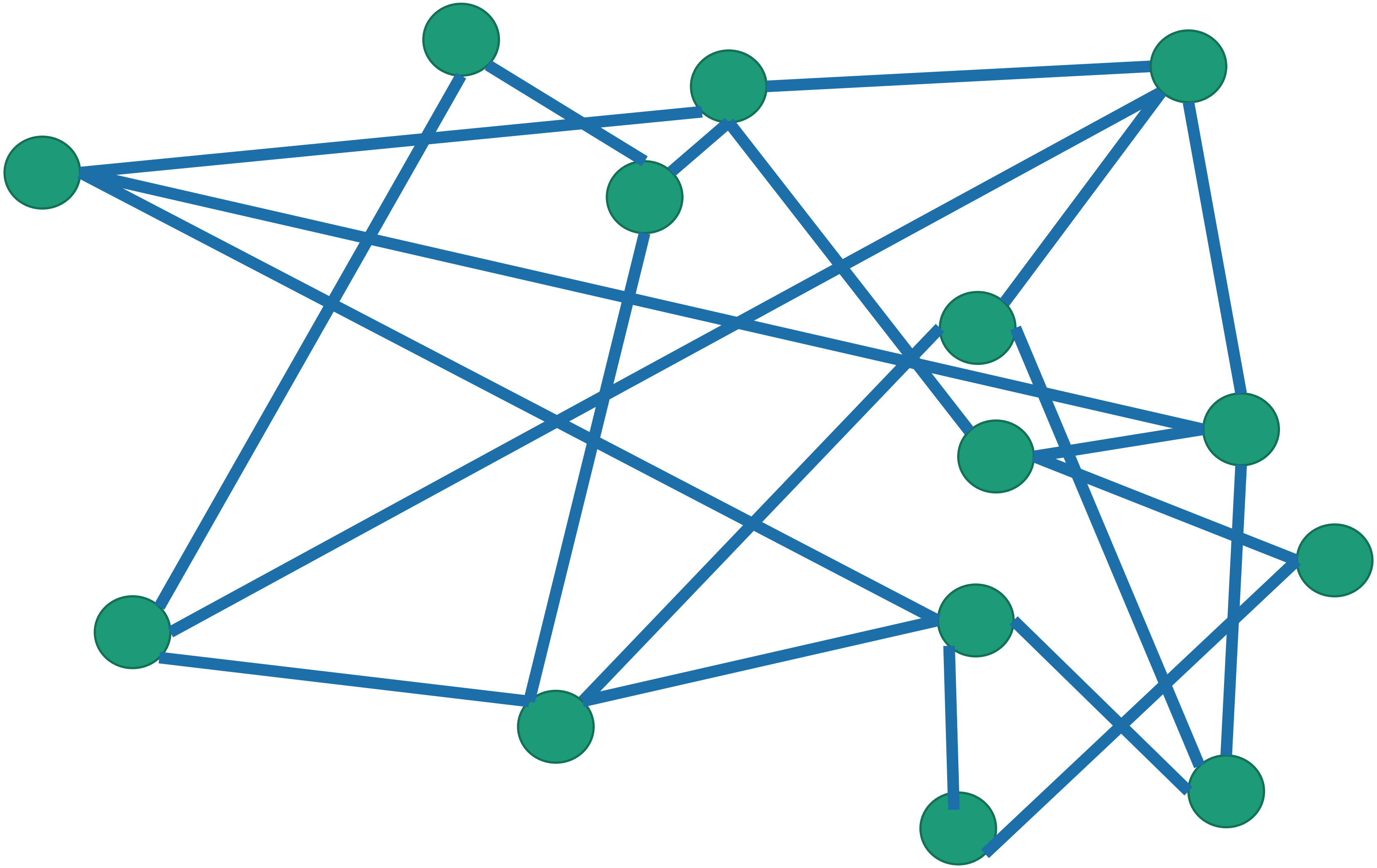
Is this graph bipartite?



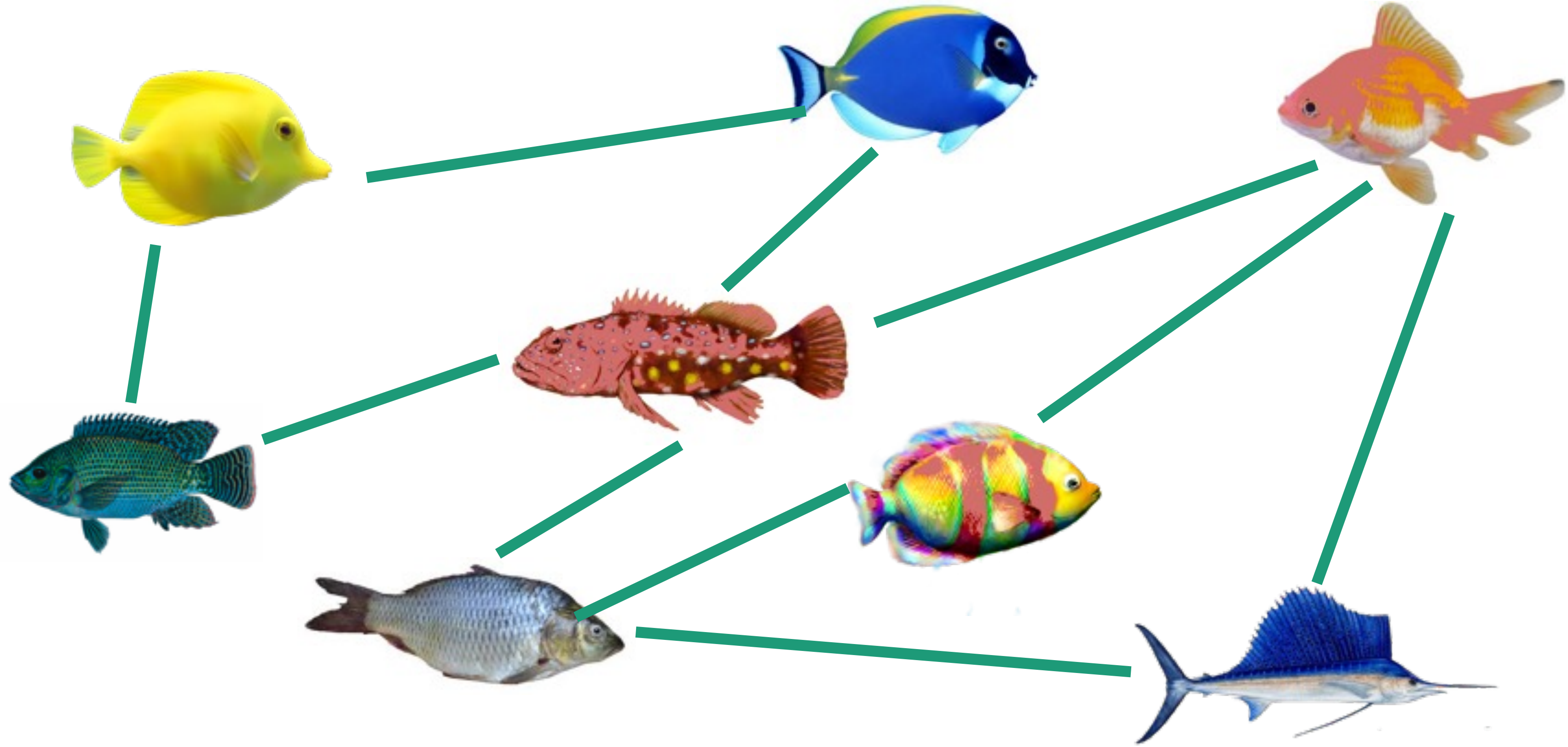
How about this one?



How about this one?



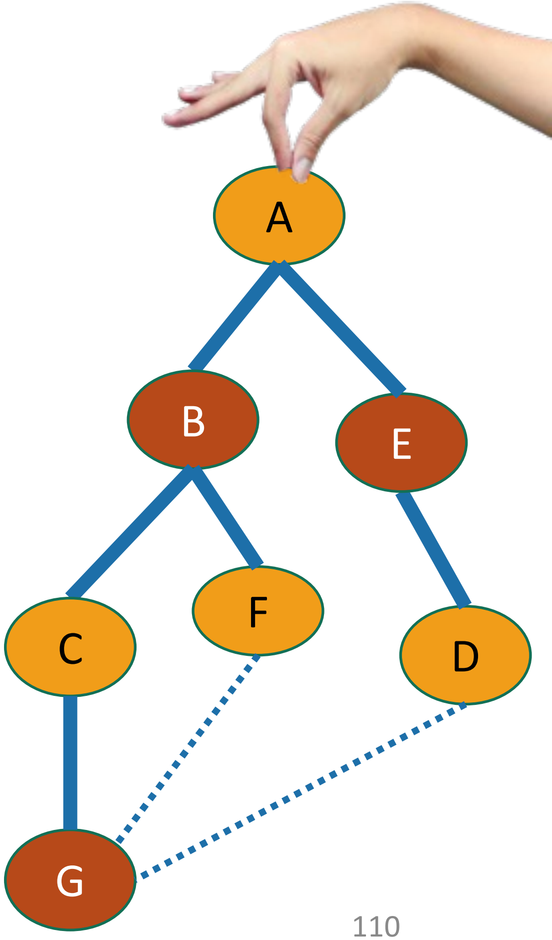
This one?



Does DFS work here too?

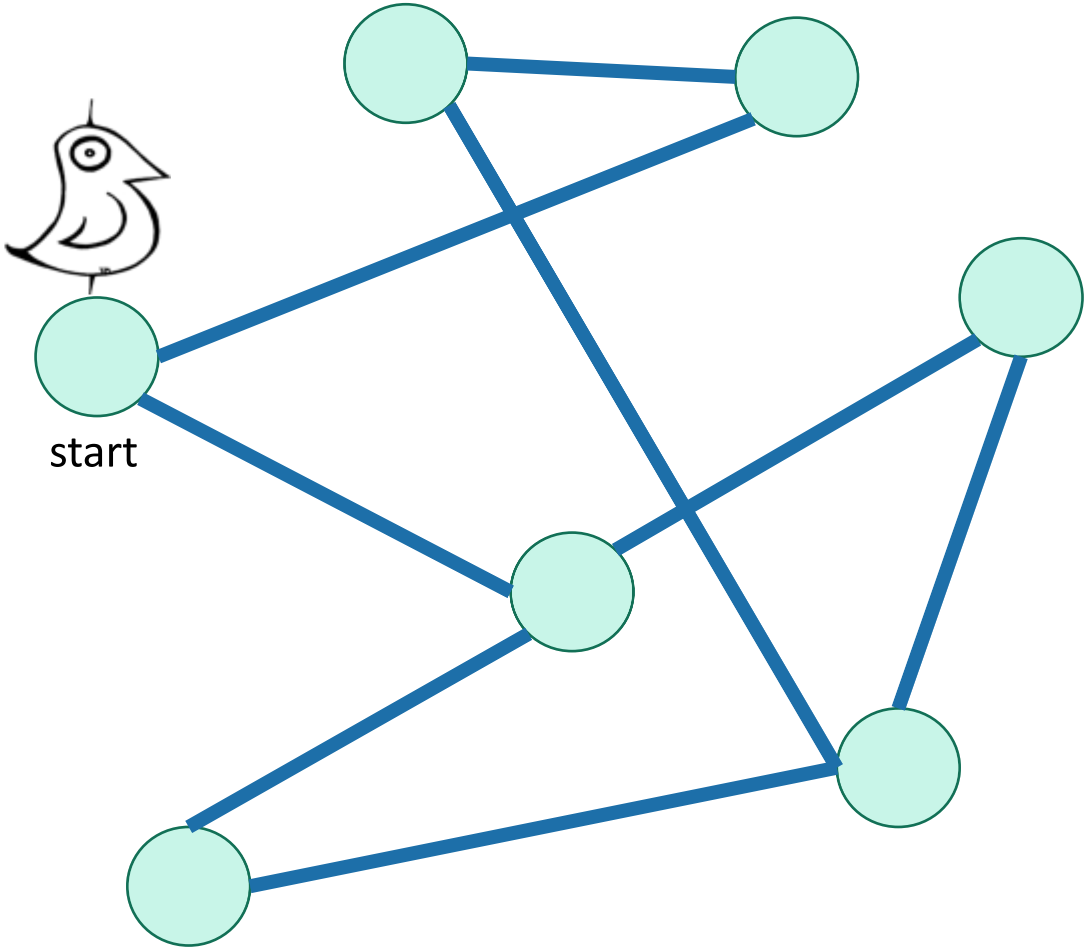
Application of BFS:
Testing Bipartiteness

- Color the levels of the BFS tree in alternating colors.
- If you never color two connected nodes the same color, then it is bipartite.
- Otherwise, it's not.



Breadth-First Search

For testing bipartite-ness



○ Not been there yet

🟠 Can reach there in zero steps

🟤 Can reach there in one step

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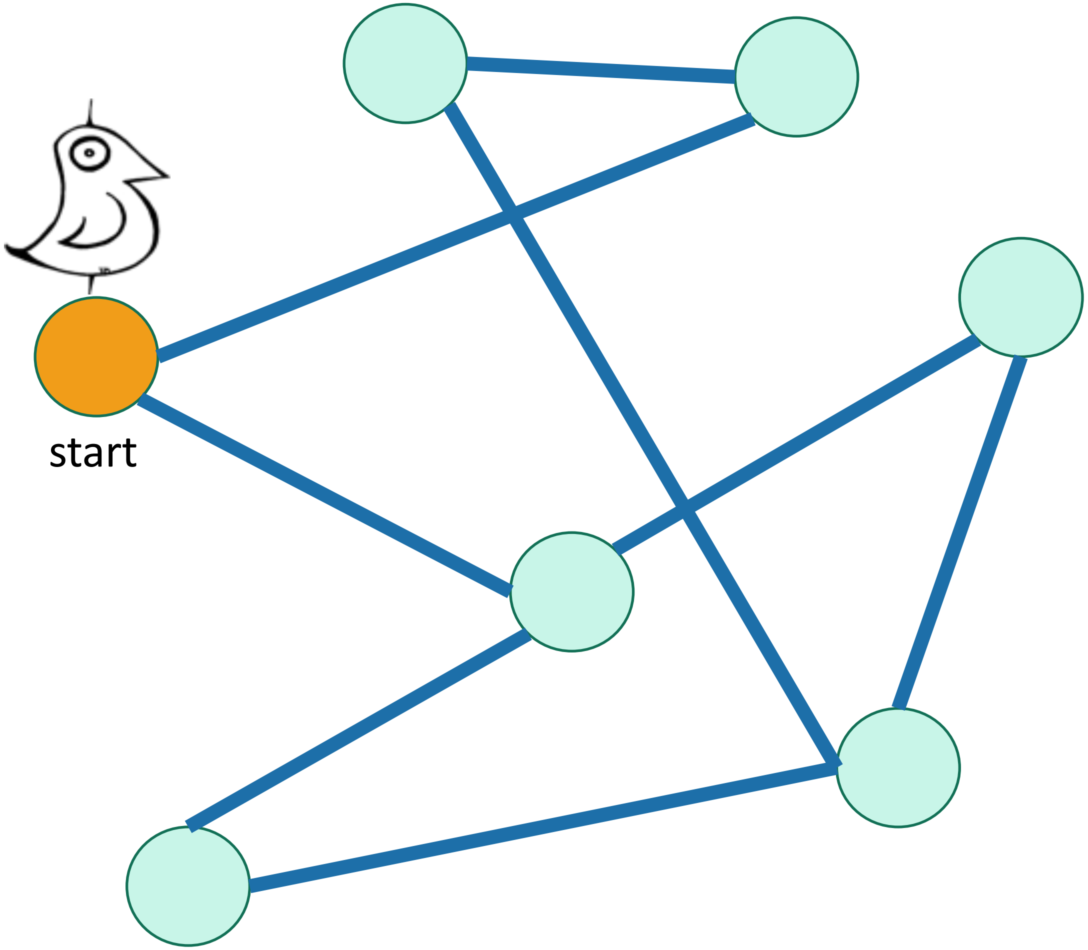
Breadth-First Search

For testing bipartite-ness



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Breadth-First Search

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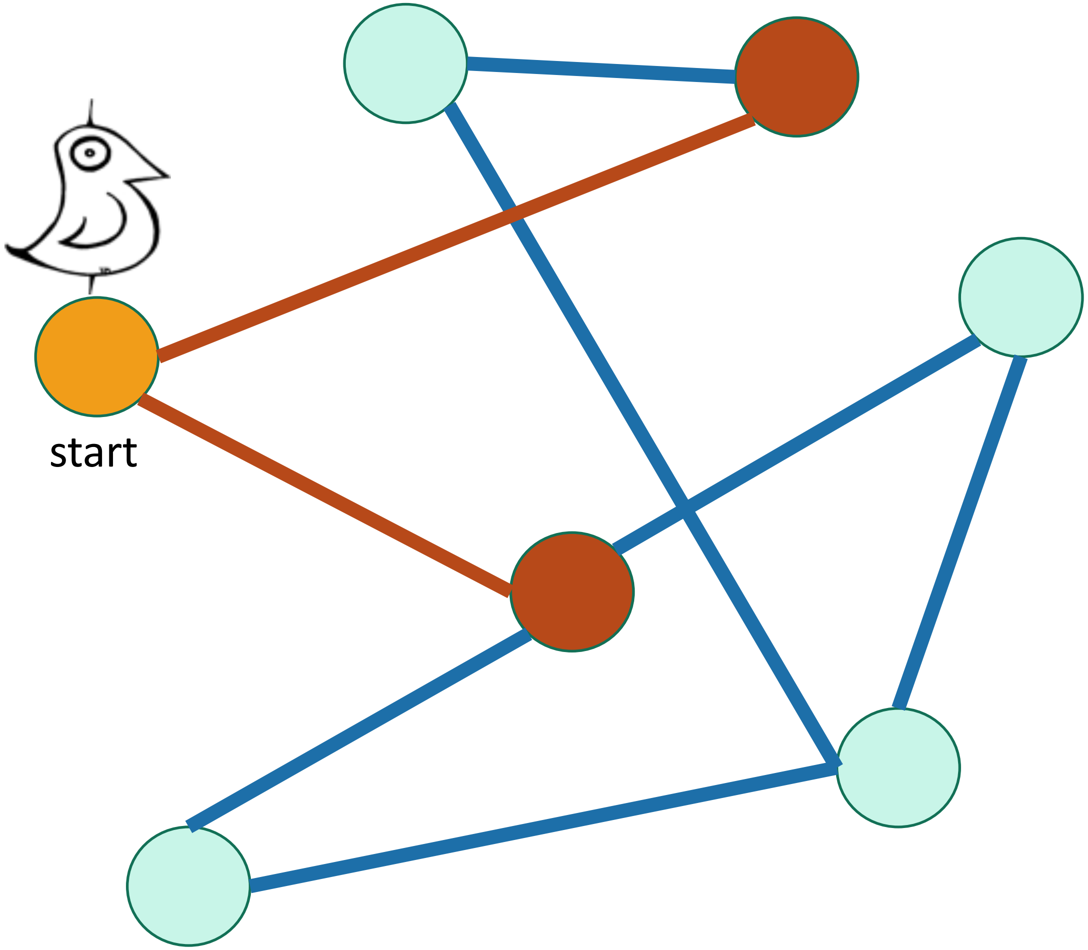

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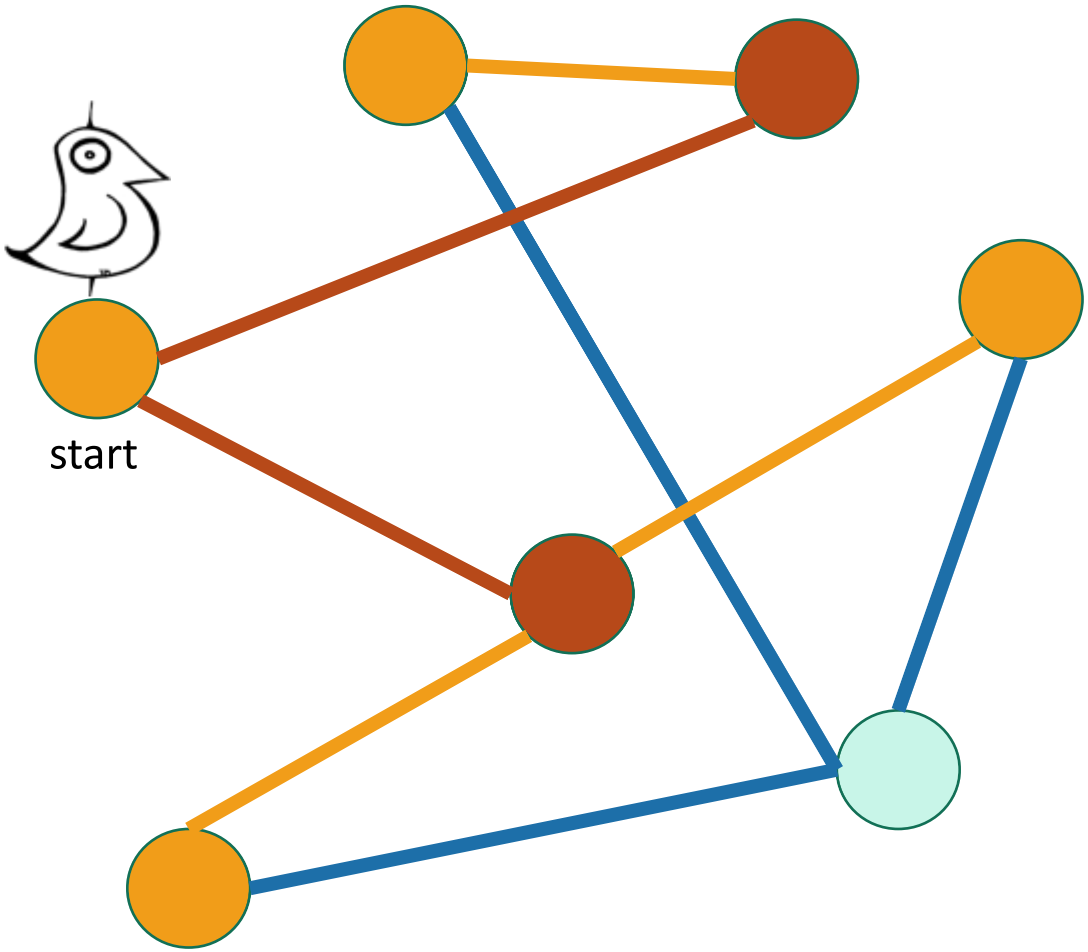
Breadth-First Search

For testing bipartite-ness



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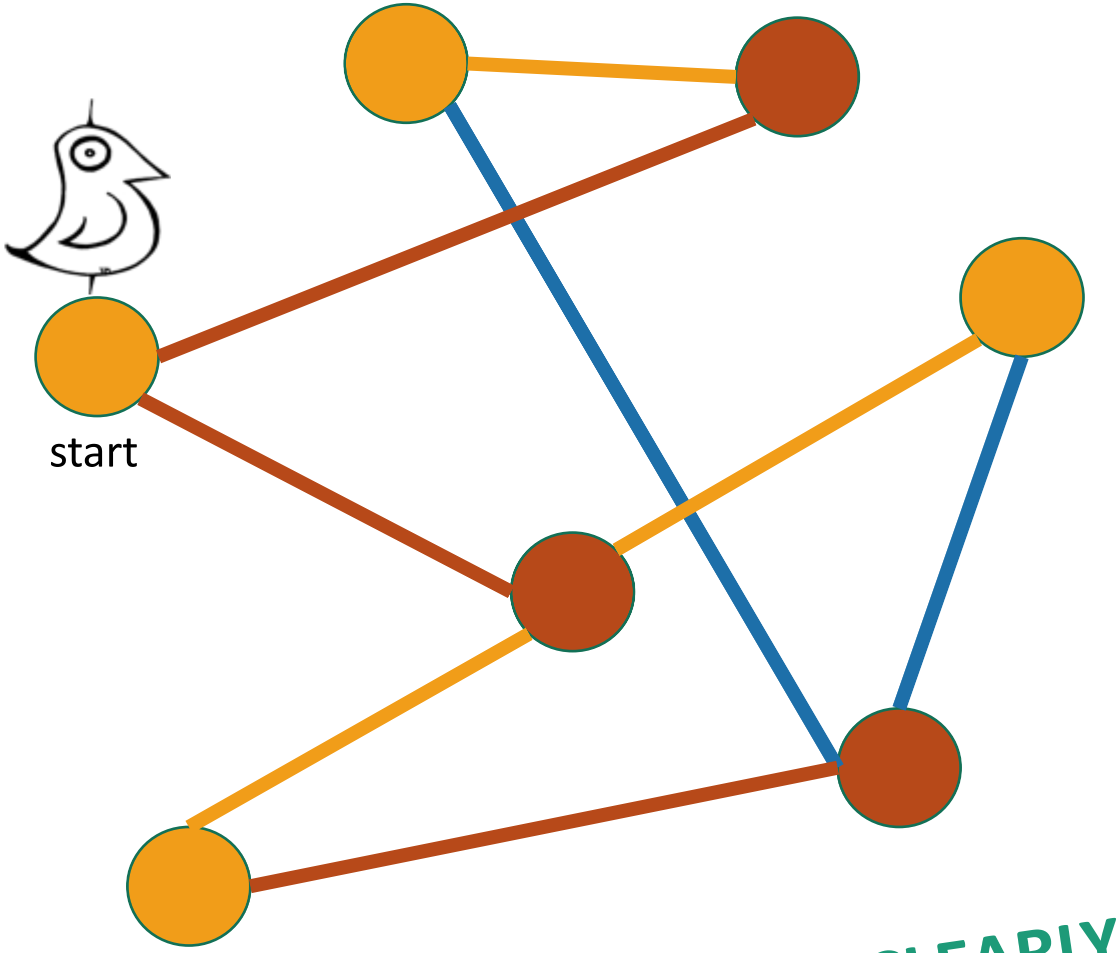

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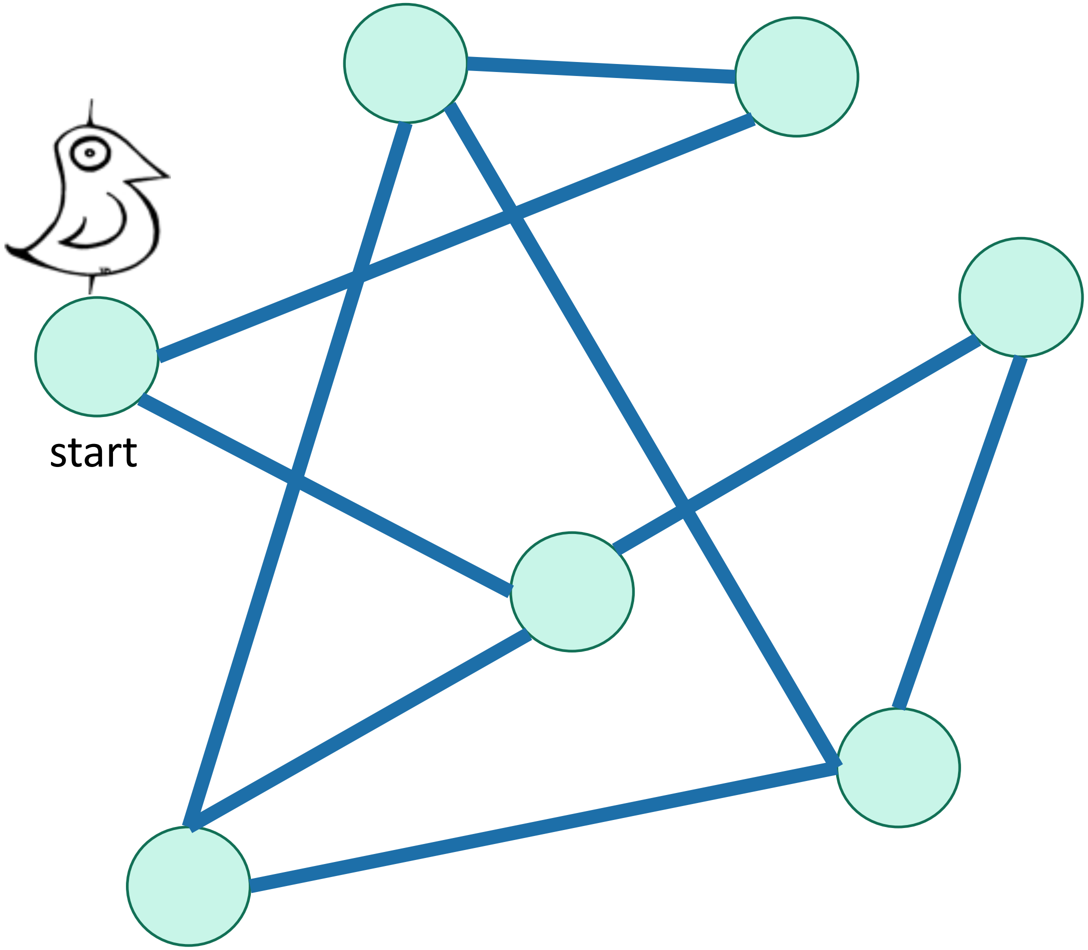
CLEARLY BIPARTITE!

Breadth-First Search

For testing bipartite-ness



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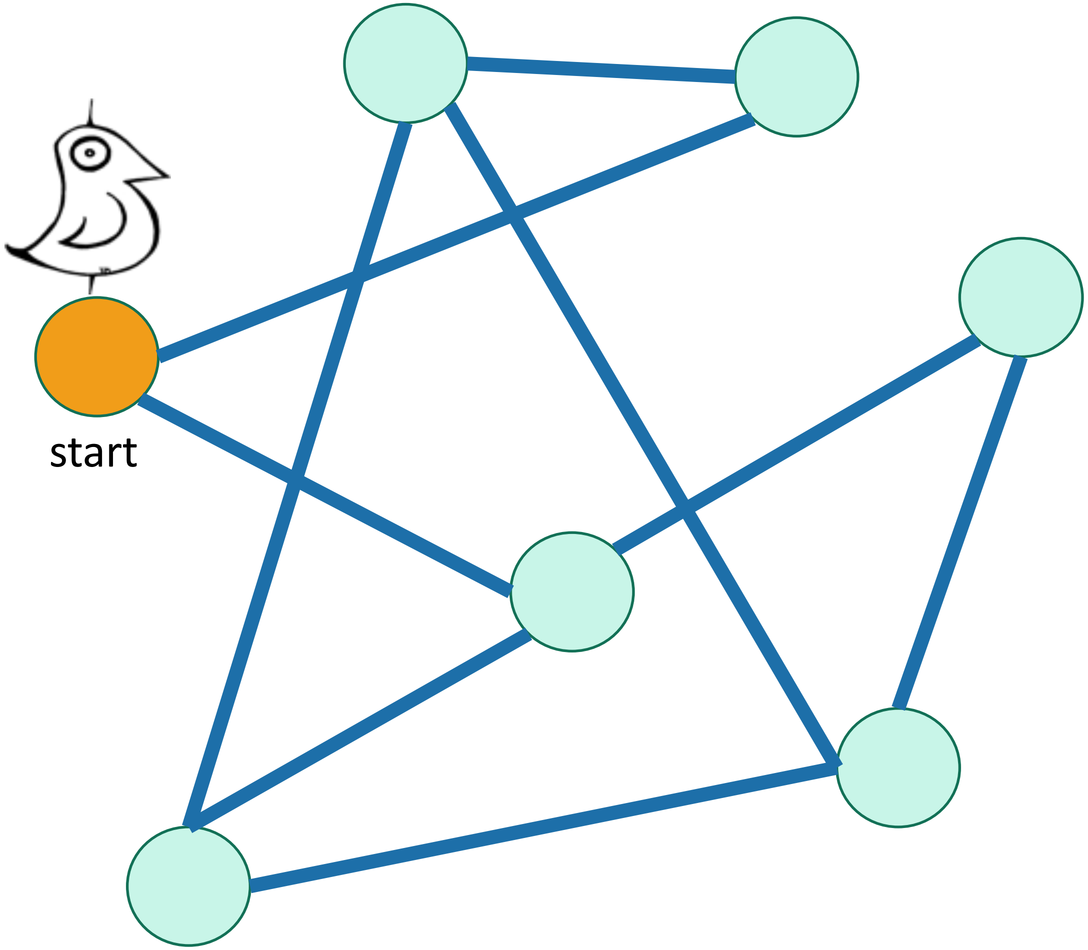
Breadth-First Search

For testing bipartite-ness



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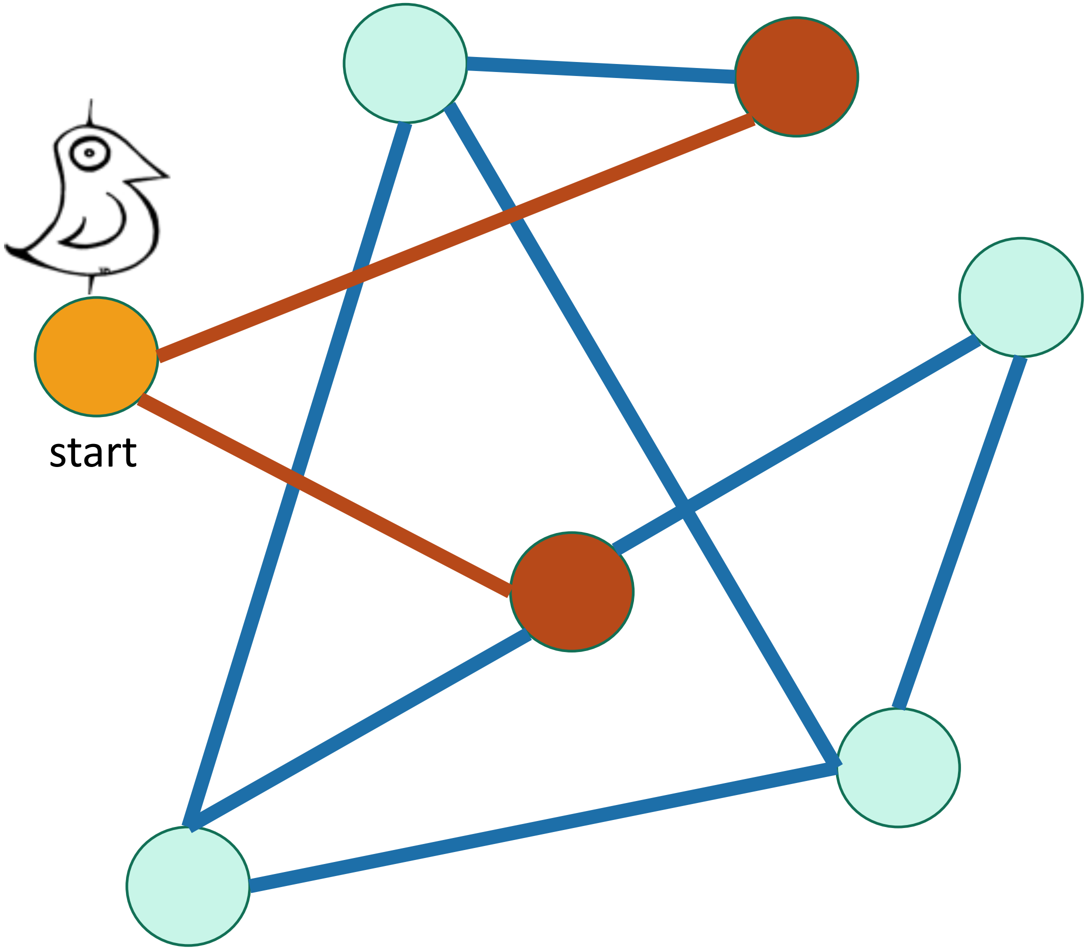

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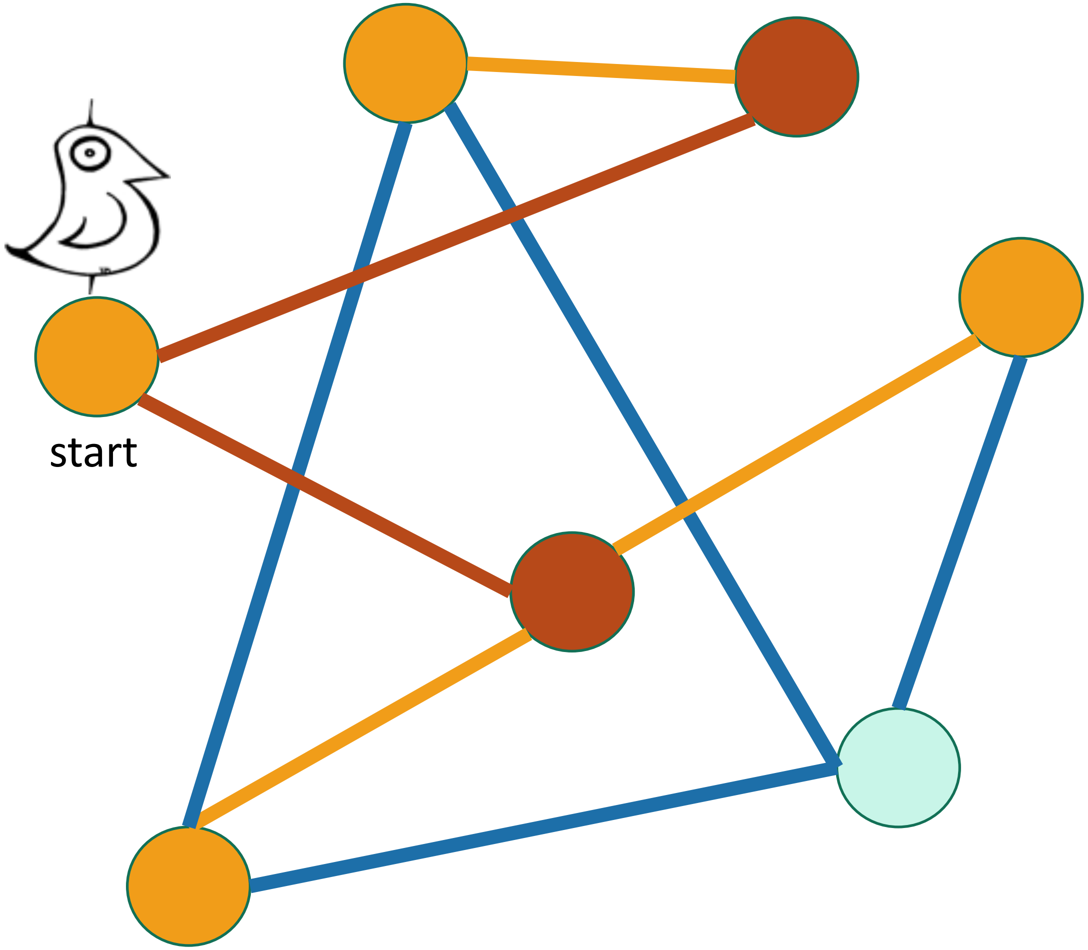

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Breadth-First Search

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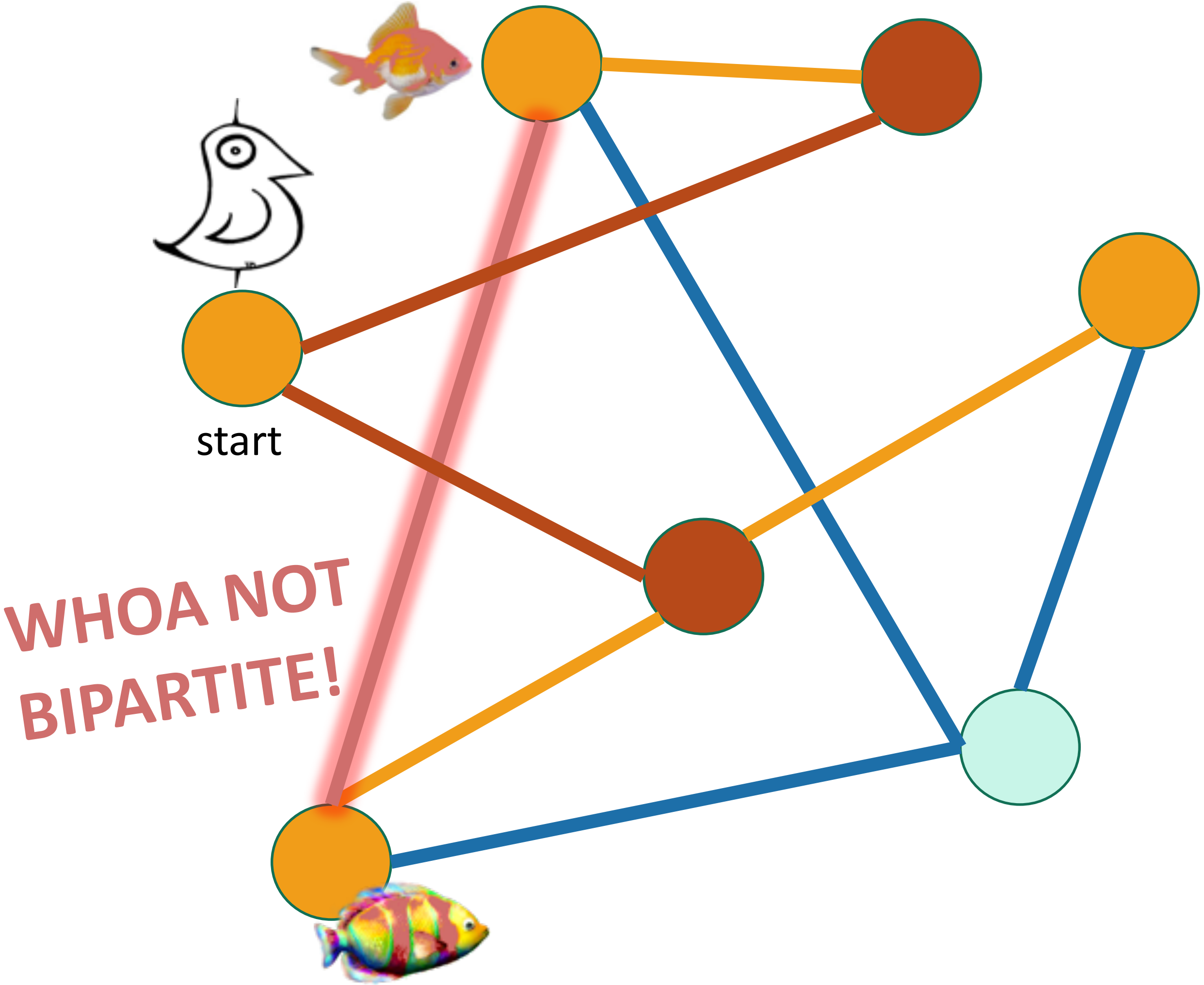

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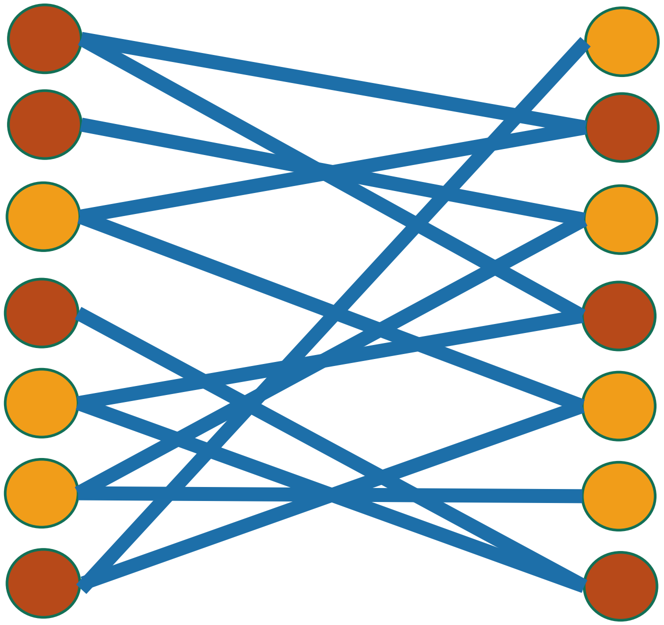
- ● Can reach there in three steps

Hang on now.

- Just because **this** coloring doesn't work, why does that mean that there is **no** coloring that works?



I can come up with plenty of bad colorings on this legitimately bipartite graph…

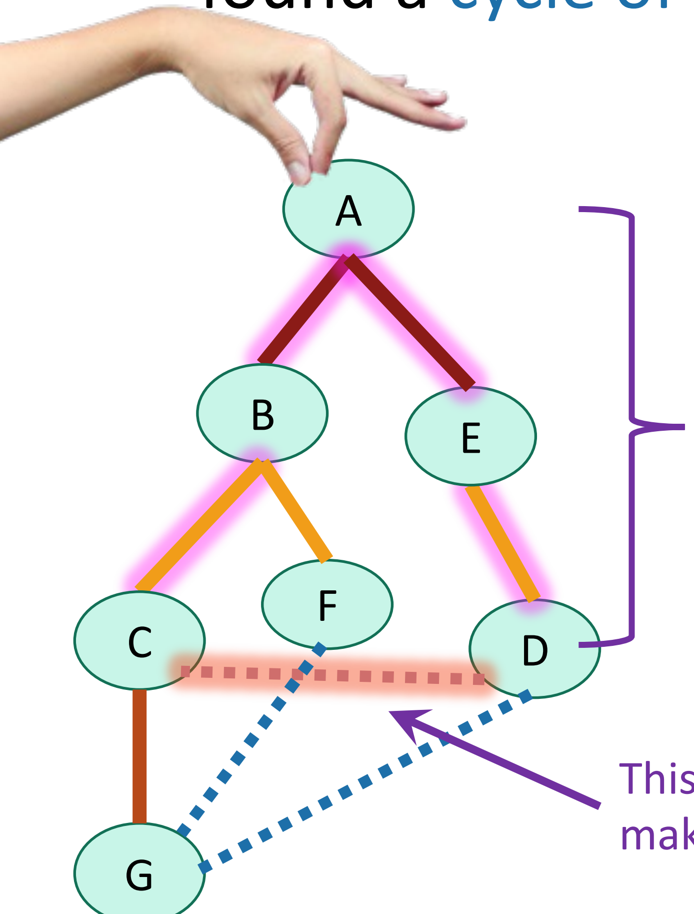
Plucky the pedantic penguin

Make this proof sketch formal!

Ollie the over-achieving ostrich

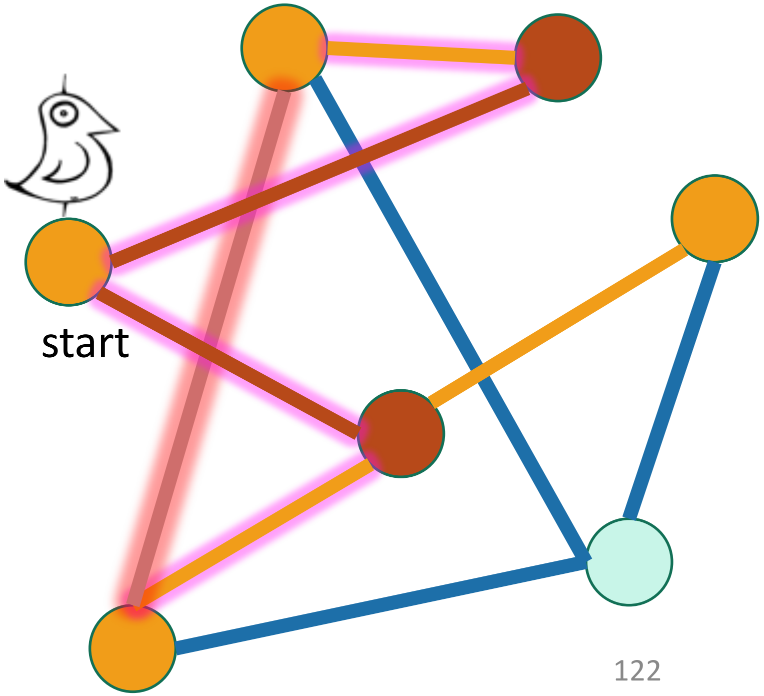
Some proof required

- If BFS colors two neighbors the same color, then it's found a cycle of odd length in the graph.



There must be an even number of these edges

This one extra makes it odd



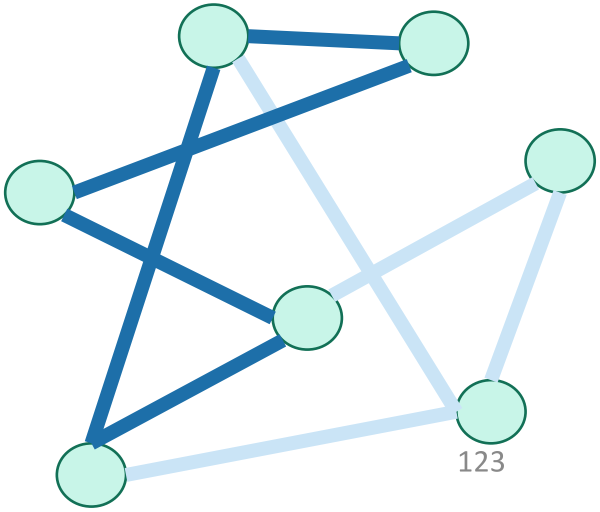
Make this proof sketch formal!

Ollie the over-achieving ostrich

Some proof required

- If BFS colors two neighbors the same color, then it's found a cycle of odd length in the graph.
- But you can **never** color an odd cycle with two colors so that no two neighbors have the same color.
  - [Fun exercise!]

- So you can't legitimately color the whole graph either.
- **Thus it's not bipartite.**



What have we learned?

BFS can be used to detect bipartite-ness in time O(n + m).

Outline

- Part 0: Graphs and terminology
- Part 1: Depth-first search
  - Application: topological sorting
  - Application: in-order traversal of BSTs
- Part 2: Breadth-first search
  - Application: shortest paths
  - Application (if time): is a graph bipartite?

Recap

Recap

- Depth-first search
  - Useful for topological sorting
  - Also in-order traversals of BSTs
- Breadth-first search
  - Useful for finding shortest paths
  - Also for testing bipartiteness
- Both DFS, BFS:
  - Useful for exploring graphs, finding connected components, etc

Still open (next few lectures)

- We can now find components in undirected graphs…
  - What if we want to find strongly connected components in directed graphs?
- How can we find shortest paths in weighted graphs?
- What is Samuel L. Jackson’s Erdos number?
  - (Or, what if I want everyone’s everyone-else number?)

Next Time

- Strongly Connected Components

Before Next Time

- Pre-lecture exercise: Strongly Connected What-Now?