## Asymptotic Notation

## 1 Definitions

Which of the following is the correct English description of $f(n)=O(g(n))$ ?
O For every constant $c>0$, there is an $n_{0}$, such that for all $n \geq n_{0}$, we have $f(n) \leq c \cdot g(n)$.
O There is some $c>0$ and some $n_{0}$, such that for all $n \geq n_{0}$ we have $f(n) \leq c \cdot g(n)$.
O For every $n_{0}$, there is some constant $c>0$ such that for all $n \geq n_{0}$ we have $f(n) \leq c \cdot g(n)$.

## Correct

Suppose that $g(n)>0$ for all integers $n$. Then is $f(n)=O(g(n))$ equivalent to the following simpler definition that avoids $n_{0}$ ? Note the implicit assumption that $f(n)$ and $g(n)$ are functions over nonnegative integers.

$$
\exists c>0: \forall n f(n) \leq c \cdot g(n)
$$

O Yes
O No

## Correct

Suppose that $f(n)=O(g(n))$. Which of the following is implied by this fact?
$\bigcirc g(n)=\Omega(f(n))$
$\bigcirc(n)=O(f(n))$
O Both
O Neither

## Correct

If $f(n)=O(g(n))$, is it true that $2^{f(n)}=O\left(2^{g(n)}\right)$ ?
O Yes
O No

## Correct

## 2 Examples

What is the smallest exponent $x$ such that

$$
n^{2}+n^{3}-n=O\left(n^{x}\right) ?
$$



Correct
Which of the following describes $n(n+1)(n+2) / 6$ ?
$O\left(n^{4}\right)$
$O\left(n^{3}\right)$
$\bigcirc \Theta\left(n^{3}\right)$
$\bigcirc \Omega\left(n^{2}\right)$
O All of the above

## Correct

For which exponents $x$ is $n(n+1) / 2=\Theta\left(n^{x}\right)$ ?
O 1
○ 2
○ 3
O All of the above

## Correct

For which function $g(n)$ is it true that $n^{2}=O(g(n))$ ?
O $g(n)=1.01^{n}$
○ $g(n)=2^{n} \cdot \sin (\pi n / 2)$
○ $g(n)=2^{n} \cdot \cos (\pi n / 2)$
O All of the above

