1 Definitions

Suppose that the nodes A, B, C in a binary search tree are arranged as follows.

Which of the following describes the relationship between A, B, C?

- A ≤ B, C
- A ≥ B, C
- A ≤ B ≤ C
- B ≤ A ≤ C

Now suppose that nodes A, B, C are arranged as follows in the binary search tree.

What is the relationship between A, B, C?

- B ≤ A ≤ C
- B ≤ C ≤ A
- C ≤ B ≤ A
- C ≤ A ≤ B

If two different binary search trees contain the same set of values, which of the following is common between them?

- Their pre-order traversals.
- Their in-order traversals.
- Their post-order traversals.
- Their root nodes.

Which of the following describes the height of a binary search tree on \( n \) nodes?

- \( O(\log n) \)
- \( \Omega(\log n) \)
- \( \Theta(\log n) \)
- All of the above.

If the length of a path from the root of a red-black tree to one of the leaf NIL nodes is 100, what could be the length of another path from the root to some other NIL node?

- 45
- 180
- 30
- All of the above.

Suppose that \( r \) is the root of a red-black tree on \( n \) nodes. Assume all nodes have distinct values. If we sort the values stored in the tree to get \( x_1 < x_2 < \cdots < x_n \), and find the index \( i \) where \( r = x_i \), what can be said about \( i \)?

- \( i \geq \Omega(n) \)
- \( i \geq \Omega(\sqrt{n}) \)
- \( i \leq 0.99n \)
- All of the above.

What is the worst-case runtime of operations INSERT/DELETE/SEARCH on a red-black tree storing \( n \) nodes?

- \( O(n) \)
- \( O(\log n) \)
- \( O(\sqrt{n}) \)
- \( O(\log^2 n) \)

2 Red-Black Trees

Is the following a valid red-black tree? We are not drawing the implicit NIL nodes.

A valid red-black tree should satisfy the following properties:

- Every node is either red or black.
- The root is black.
- All leaves (NIL nodes) are black.
- If a node is red, both its children are black.
- Every path from a node to its descendant leaf nodes contains the same number of black nodes.

Yes
No

Is the following a valid red-black tree?

Yes
No

Is the following a valid red-black tree?

Yes
No

Which of the following describes the height of a red-black tree on \( n \) nodes?

- \( O(\log n) \)
- \( O(\log^2 n) \)
- \( O(\log n^2) \)
- All of the above.

If the length of a path from the root of a red-black tree to one of the leaf NIL nodes is 100, what could be the length of another path from the root to some other NIL node?

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- \( i \leq 0.99n \)
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What is the worst-case runtime of operations INSERT/DELETE/SEARCH on a red-black tree storing \( n \) nodes?

- \( O(n) \)
- \( O(\log^2 n) \)
- \( O(\log n^2) \)
- \( O(\sqrt{n}) \)

Correct