1 Max-Flow

For each of the statements below, select whether it is possible or not.

In a residual graph $G$, we have two vertices $u$ and $v$ such that the sum of capacities on the edge from $u$ to $v$ and the edge from $v$ to $u$ is less than the initial capacity of the edge between a $u$ and $v$.

Possible

Impossible

Correct

There can be an instance of max-flow where some of the outgoing edges from the source $S$ and some of the incoming edges to the sink $T$ are set to their full capacity.

Possible

Impossible

Correct

There exists a vertex $v$ other than the source and the sink where its incoming flow is less than its outgoing flow.

Possible

Impossible

Correct

Suppose we have a graph with integer capacities. Can we always find a single edge and increase its capacity by 1 to make the value of max-flow increase by 1?

Always possible

Always impossible

Sometimes possible, and sometimes impossible

Correct

Suppose we have a graph with integer capacities and there is no maximum flow between a source and a sink vertex. We want to increase the capacity of some of the edges in order to increase the max-flow by 1. Do we need to increase any edge's capacity by 2 or more units?

Always possible while increasing edge capacities by at most 1

Sometimes possible while increasing edge capacities by at most 1

Always impossible

Correct

For each of the statements below, select whether it is possible or not.

There exists a vertex other than the source and the sink where its incoming flow is less than its outgoing flow.

Possible

Impossible

Correct

There can be an instance of max-flow where some of the outgoing edges from the source $S$ and some of the incoming edges to the sink $T$ are set to their full capacity.

Possible

Impossible

Correct

There exists a vertex $v$ other than the source and the sink where its incoming flow is less than its outgoing flow.

Possible

Impossible

Correct

2 Max-Flow Example

We start with a graph that has no incoming edges to the source $S$ and no outgoing edges from the sink $T$. The residual graph is pictured below.

What is the current flow from $S$ to $T$?

Possible

Impossible

Correct

What is the value of max-flow?

Possible

Impossible

Correct

What is the only remaining augmenting path in this residual graph? Write it as a string of vertices in increasing order from the source to the sink.

Possible

Impossible

Correct

3 Matching

The company Algorithmia is preparing to transition back to the office in-person work. They have a office with $n$ employees that they need to assign to these offices. Given COVID restrictions, each office has a capacity $C_j$.

Each employee has a list of offices that are within their driving distance. Algorithmia wants to assign as many employees to offices as possible, as long as, every employee is assigned to a single office which in within the employee's driving distance, and the number of employees assigned to every office is less than or equal to the office's capacity.

You are asked to find the maximum possible number of employees that can be assigned to the offices. Given COVID restrictions, each office has a capacity $C_j$.

Is the current flow the max-flow?

Yes

No

Correct

The office $j$'s capacity

Possible

Impossible

Correct

All of the above are valid.

Correct

4 Distinct Flows

Suppose that we have a graph on $n$ nodes with capacities on the edges. For each pair of nodes $u$ and $v$ compute the maximum flow with $u$ as a source and $v$ as a sink. In total we collect $O(n^2)$ numbers. If $n$ is the minimum among these $O(n^2)$ numbers, how many times do you need to repeat this test?

Choose the smallest correct range.

$O(\log n)$ times.

$O(n)$ times.

$O(n^2)$ times.

Correct