1 Parts of Minimum Spanning Tree

For each of the following edges, determine whether it has to be necessarily part of some minimum spanning tree.

- The minimum edge coming out of a vertex is always part of some MST.
- An edge on the shortest path between two vertices is not necessarily part of some MST.
- The smallest edge going across a cut is not necessarily part of some MST.

2 Minimum Spanning Tree Example

Consider the graph below:

What is the weight of the minimum spanning tree in this graph?
- 16

Assume we run Prim's algorithm on this graph to find the minimum spanning tree starting at vertex A.

What is the weight of the first edge added?
- 4

What is the weight of the second edge added?
- 1

What is the weight of the third edge added?
- 2

What is the weight of the fourth edge added?
- 3

What is the weight of the fifth edge added?
- 6

Assume we run the Kruskal's algorithm on this graph to find the minimum spanning tree.

What is the weight of the first edge added?
- 1

What is the weight of the second edge added?
- 2

What is the weight of the third edge added?
- 3

What is the weight of the fourth edge added?
- 4

What is the weight of the fifth edge added?
- 6

3 Maximum Spanning Tree

Can we find the maximum spanning tree (instead of minimum) using the same Kruskal or Prim algorithms?

- Yes, we can multiply the weights by \(-1\) and run the minimum spanning tree algorithms.
- Yes, we can modify both algorithms by choosing the edge with the greatest weight each time (instead of the edge with the least weight).
- Both of the above are correct.
- No, we can't.

Consider the graph from the previous problem. This time we want to find the maximum spanning tree.

What is the weight of the maximum spanning tree in the graph?
- 30

Assume that we run Prim's algorithm on this graph to find the maximum spanning tree starting at vertex A.

What is the weight of the first edge added?
- 7

What is the weight of the second edge added?
- 4

What is the weight of the third edge added?
- 5

What is the weight of the fourth edge added?
- 8

What is the weight of the fifth edge added?
- 6

Assume we run Kruskal's algorithm on this graph to find the maximum spanning tree.

What is the weight of the first edge added?
- 8

What is the weight of the second edge added?
- 7

What is the weight of the third edge added?
- 6

What is the weight of the fourth edge added?
- 5

What is the weight of the fifth edge added?
- 4

Note: the answer is given as a specific sum of the weights of the selected edges, not as a graph. The graph is not shown here. 