In the select algorithm, the runtime is represented with the recurrence relation

\[ T(n) = O(n) + T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right) . \]

Here, \( T\left(\frac{n}{5}\right) \) is for selecting the pivot, and \( T\left(\frac{7n}{10}\right) \) is for the recursive call to select the \( k \)-th element.

Consider the modified version of the select algorithm, where we split our array into \( \left\lceil \frac{n}{7} \right\rceil \) groups of size \( \leq 7 \) instead. What would be the recurrence relation for this modified version? Specifically, if we write the recurrence relation as \( T(n) = O(n) + T\left(\frac{n}{7}\right) + T\left(\frac{7n}{10}\right) \), where \( a, b, \) and \( c \) are non-negative integers, what are the smallest possible values of \( a, b, \) and \( c \)?

\[
\begin{align*}
a &= 7 & \text{Correct} \\
b &= 5 & \text{Correct} \\
c &= 7 & \text{Correct}
\end{align*}
\]

What is the smallest exponent \( x \) such that the modified version of the select described above on an array of size \( n \) always takes time \( O(n^x) \)?

\[
\begin{align*}
x &= 1 & \text{Correct}
\end{align*}
\]

Now assume that the \( O(n) \) work per recursive step takes exactly \( n \) units of time on our machine. In other words, suppose that the recurrence relation for the runtime is

\[ T(n) = n + T\left(\frac{n}{7}\right) + T\left(\frac{7n}{10}\right) . \]

What is the smallest coefficient \( C \) such that we can use the substitution method to prove that the recurrence relation for the modified select algorithm is \( T(n) \leq Cn \)?

\[
\begin{align*}
C &= 7 & \text{Correct}
\end{align*}
\]

Now consider another modified version of the select algorithm, where we split our array into \( \left\lceil \frac{n}{3} \right\rceil \) groups of size \( \leq 3 \) instead. What would be the recurrence relation for this modified version? Specifically, if we write the recurrence relation as

\[ T(n) = n + T\left(\frac{n}{3}\right) + T\left(\frac{bn}{c}\right) . \]

where \( a, b, \) and \( c \) are non-negative integers, what are the smallest possible values of \( a, b, \) and \( c \)?

\[
\begin{align*}
a &= 3 & \text{Correct} \\
b &= 2 & \text{Correct} \\
c &= 3 & \text{Correct}
\end{align*}
\]

Which one is true for the modified select recurrence relation that you came up with in the last part?

- \( T(n) = \Theta(n) \) \[ \circ \]  
- \( T(n) = \Theta(n \log n) \) \[ \circ \]  
- \( T(n) = \Theta(n^2) \) \[ \text{Correct} \]