

# Selection

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In the select algorithm, the runtime is represented with the recurrence relation

$$T(n) = O(n) + T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right).$$

Here,  $T\left(\frac{n}{5}\right)$  is for selecting the pivot, and  $T\left(\frac{7n}{10}\right)$  is for the recursive call to select the  $k$ -th element.

Consider the modified version of the select algorithm, where we split our array into  $\lceil \frac{n}{7} \rceil$  groups of size  $\leq 7$  instead. What would be the recurrence relation for this modified version? Specifically, if we write the recurrence relation as  $T(n) = O(n) + T\left(\frac{n}{a}\right) + T\left(\frac{bn}{c}\right)$ , where  $a$ ,  $b$ , and  $c$  are non-negative integers, what are the smallest possible values of  $a$ ,  $b$ , and  $c$ ?

$a =$

Correct

$b =$

Correct

$c =$

Correct

What is the smallest exponent  $x$  such that the modified version of the select described above on an array of size  $n$  always takes time  $O(n^x)$ ?

Correct

Now assume that the  $O(n)$  work per recursive step takes exactly  $n$  units of time on our machine. In other words, suppose that the recurrence relation for the runtime is

$$T(n) = n + T\left(\frac{n}{a}\right) + T\left(\frac{bn}{c}\right).$$

What is the smallest coefficient  $C$  such that we can use the substitution method to prove that the recurrence relation for the modified select algorithm is  $T(n) \leq Cn$

Correct

Now consider another modified version of the select algorithm, where we split our array into  $\lceil n/3 \rceil$  groups of size  $\leq 3$  instead. What would be the recurrence relation for this modified version? Specifically, if we write the recurrence relation as

$$T(n) = n + T\left(\frac{n}{a}\right) + T\left(\frac{bn}{c}\right),$$

where  $a$ ,  $b$ , and  $c$  are non-negative integers, what are the smallest possible values of  $a$ ,  $b$ , and  $c$ ?

$a =$

Correct

$b =$

Correct

$c =$

Correct

Which one is true for the modified select recurrence relation that you came up with in the last part?

- $T(n) = \Theta(n)$
- $T(n) = \Theta(n \log n)$
- $T(n) = \Theta(n^2)$

Correct