

Pre-lecture exercises will not be collected for credit. However, you will get more out of each lecture if you do them, and they will be referenced during lecture. We recommend writing out your answers to pre-lecture exercises before class. Pre-lecture exercises usually should not take you more than 30 minutes.

## Pre-Lecture Exercises

In this pre-lecture exercise, we will recall a little bit of probability, random variables and expectations. Hopefully, you are familiar with this, because you will need it in lecture 5!

1. Let  $X$  be a random variable which is 1 with probability  $1/100$  and 0 with probability  $99/100$ .
  - (a) What is the expected value  $\mathbb{E}[X]$ ?
  - (b) Suppose you draw  $n$  independent random variables,  $X_1, X_2, \dots, X_n$ , distributed like  $X$ . What is the expected value  $\mathbb{E}[\sum_{i=1}^n X_i]$ ?
  - (c) Suppose we draw independent random variables  $X_1, X_2, \dots$  and we stop when we see the first 1. For example, if we draw

$$X_1 = 0, X_2 = 0, X_3 = 0, X_4 = 1, \dots$$

then we would stop at  $X_4$ . Let  $N$  be the last index that we draw; so in the previous example,  $N = 4$ . How big do you expect  $N$  to be?

**Note:** Actually figuring out  $\mathbb{E}[N]$  from scratch is a bit tricky, although you may have seen it in CS 109. But even if you don't do it rigorously, intuitively how big do you expect  $N$  to be?

2. Consider the following pseudocode, which sorts an array  $A$ . Someone else has implemented the random shuffling for us, so do not worry about the details of that procedure.

```
BogoSort(A) :
while  $A$  is not sorted do
  | randomly shuffle  $A$ 
return  $A$ 
```

- (a) Let  $X_i$  be a random variable which is 1 if  $A$  is sorted after the  $i$ -th call, and 0 otherwise.
- (b) What is  $\mathbb{E}[X_i]$ ?
- (c) What is the *expected* number of times that BogoSort executes the while loop?