CS 161 (Stanford, Winter 2025) Section 7

1 Warm-up: Greedy or Not?

Sometimes it can be tricky to tell when a greedy algorithm applies. For each problem, say whether or not the greedy solution would work for the problem. If it wouldn't work, give a counter example.

1. You have unlimited objects of different sizes, and you want to completely fill a box with as few objects as possible, or output that it is impossible. (Greedy: Keep putting the largest object possible in for the space you have left)
2. You have unlimited objects of size 3^k for every k, and you want to completely fill a box with as few objects as possible. (Greedy: same approach as the previous problem)
3. You have lines that can fit a fixed number of characters. You want to print out a series of words in a given order while using as few lines as possible. (Greedy: Fit as many words as you can on a given line)
4. There are n hotels in a line, each distance 1 apart and hotel i costing h_i dollars to stay at. You can travel at most distance k every day. Find the minimum total cost of hotels you need to stop at. (Greedy: Go as far as you can before stopping at a hotel)
5. There are r ropes of different lengths, and the cost of connecting two of them is the sum of their lengths. Find a way to connect all ropes into a single rope with the minimum total cost. (Greedy: repeatedly add the smallest two values).

2 Encoding

Suppose we encode lowercase letters into a numeric string as follows: we encode a as 1, b as 2, ..., and z as 26. Given a numeric string S of length n, develop an $O(n)$ algorithm to find how many letter strings this can correspond to. For example, for the numeric string 123, the algorithm should output 3 because the letter strings that map to this numeric string are abc (decoded as "1", "2", "3") lc, (decoded as "12", "3") and aw (decoded as "1", "23").

3 Knight Moves

Given an 8×8 chessboard and a knight that starts at position $a1$, devise an algorithm that returns how many ways the knight can end up at position xy after k moves. Knights move ± 1 squares in one direction and ± 2 squares in the other direction. In other words, knights move in a pattern similar to a L.

Note: on a chessboard, rows are labeled from 1 to 8 and columns are labeled from *a* to *h*, as seen below.

4 String Cutting

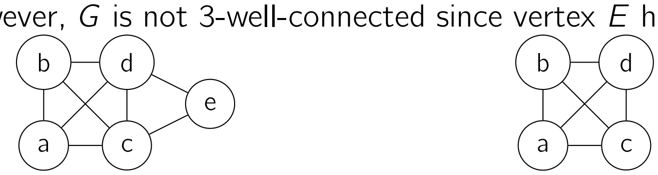
Suppose we have a string of length k, where k is a positive integer. We would like to cut the string into integer-length segments such that we maximize the *product* of the resulting segments' lengths. Multiple cuts may be made. For example, if $k = 8$, the maximum product is 18 from cutting the string into three pieces of length $3, 3$, and 2. Write an algorithm to determine the maximum product for a string of length k.

5 Well Connected Graphs

Let $G = (V, E)$ be an undirected, unweighted graph. For a subset $S \subseteq V$, define the **subgraph induced by** S to be the graph $G' = (S, E')$, where $E' \subseteq E$, and an edge $\{u, v\} \in E$ is included in E' if and only if $u \in S$ and $v \in S$.

For any $k < n$, say that a graph G is k-well-connected if every vertex has degree at least k. (That is, if there are least k edges coming out of each vertex).

For example, in the graph G below, the subgraph G' induced by $S = \{a, b, c, d\}$ is shown on the right. G' is 3-well-connected, since every vertex in G' has degree at least 3. However, G is not 3-well-connected since vertex E has degree 2.



$G = (V, E)$ $\quad$ $G' = (S, E')$, for $S = \{a, b, c, d\}$

Observation: If G' is a k-well-connected subgraph induced by S, and $v \in V$ has degree $< k$, then $v \notin S$. This is because v would have degree $< k$ in the induced subgraph G' as well, and so G' couldn't be k-well-connected if it included v.

Guided by the **observation** above, design a greedy algorithm to find a maximal set $S \subseteq V$ so that the subgraph $G' = (S, E')$ induced by S is k-well-connected. You do not need to formally prove why your algorithm is correct, but give an informal but convincing justification.

In the example above, if $k = 3$, your algorithm should return $\{a, b, c, d\}$, and if $k = 4$ your algorithm should return the empty set.

You may assume that your representation of a graph supports the following operations:

- `degree(v)`: return the degree of a vertex in time $O(1)$
- `remove(v)`: remove a vertex and all edges connected to that vertex from the graph, in time $O(\text{degree}(v))$.

Your algorithm should run in time $O(n^2)$.

6 Mice to Holes

There are n mice and n holes along a line. Each hole can accommodate only 1 mouse. A mouse can stay at his position, move one step right from x to $x + 1$, or move one step left from x to $x - 1$. Any of these moves consumes 1 minute. Mice can move simultaneously. Given initial mice positions and constant hole positions, design an algorithm that assigns mice to holes such that the time it takes for the last mouse to get to a hole is minimized, and return the amount of time it takes for that last mouse to get to its hole. Justify the correctness of this algorithm.

Example:

Mice positions: $[4, -4, 2]$

Hole positions: $[4, 0, 5]$

Best case: the last mouse gets to its hole in 4 minutes. $\{4 \to 4, -4 \to 0, 2 \to 5\}$ and $\{4 \to 5, -4 \to 0, 2 \to 4\}$ are both possible solutions.