CS 161 (Stanford, Winter 2026) Prereq Quiz

Exercises. The following questions cover material that we will use going forward in CS 161. Your work will not be graded and you are not required to complete all problems in detail, but you should make sure you are comfortable with all concepts used here. You are encouraged to ask for help on Ed or in office hours.
Note: many of these problems can be solved using more than one method. If your solution looks different than the official answer, it does not mean that you are wrong. If you aren't sure of your answer, feel free to post on Ed or ask during office hours.

1 Induction

1.1 Sums of squares

Show that for all $n \geq 1$,

$$1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

1.2 Fibonacci parity

The Fibonacci numbers are defined by $F(0) = 0$, $F(1) = 1$, and $F(n) = F(n-1) + F(n-2)$ for $n \geq 2$. Show that every third Fibonacci number is even.

1.3 Sums of cubes

Show that for all $n \geq 1$, $1^3 + 2^3 + \ldots + n^3$ is a perfect square.

1.4 Dividing chocolate

Consider a chocolate bar made up of squares in an $n \times m$ grid pattern. Show that it takes $nm - 1$ breaks to break the bar completely into 1×1 squares.

1.5 Friendship parity

Consider a group of n people where some pairs of people are friends with each other. (For example, in a group of Alice, Bob, and Carol, perhaps Alice and Bob are friends, and Alice and Carol are friends, but Bob and Carol are not friends.) Show that there is an even number of people who have an odd number of friends.

1.6 Coin values

Suppose a country only has coins of value 3 and 5. Show that it's possible to pay for any value that is at least 8.

1.7 Coin flip parity

Show that if a fair coin is flipped n times, the number of heads is equally likely to be even or odd.

1.8 Binary search

Suppose we are using binary search to find a given number in a sorted list of n numbers. Show that if $n \leq 2^k - 1$, then we will need to access at most k elements of the list.

2 Probability

2.1 Coin flips 1

Flip a fair coin until it lands on heads, and let T be the total number of times the coin was flipped. What is the expected value of T?

(a) 1 (b) 2 (c) e (d) Undefined

2.2 Coin flips 2

Flip n fair coins. Gather any coins that landed on tails and flip them again. Repeat until every coin has landed on heads. What is the expected total number of coin flips you will complete?

(a) $2n$ (b) $n \log n$ (c) $\frac{1}{2}n^2$ (d) Undefined

2.3 Coin flips 3

Flip a coin until it lands on heads. Starting with \$1 on the table, each time the coin lands on tails, double the amount of money on the table. When you flip heads for the first time, collect all the money on the table. If it costs \$2 to play this game, what is the expected amount of money you will earn?

(a) -1 (b) 0 (c) 2 (d) Undefined

2.4 Conditional expectation

Let X and Y be the results of rolling two standard six-sided dice. What is the expected value of X given that the sum of the dice is 9?

(a) 3.5 (b) 4 (c) 4.5 (d) 5

2.5 Counting 1

Consider putting three identical balls into 10 numbered bins. Out of all possible configurations, in approximately what fraction is each ball in a different bin?

(a) 0.55 (b) 0.67 (c) 0.72 (d) 0.93

2.6 Counting 2

Put each of three balls independently into one of 10 bins uniformly at random. What is the approximate probability that each ball is in a different bin?

(a) 0.55 (b) 0.67 (c) 0.72 (d) 0.93

2.7 Conditional probability

There are 300 students enrolled in CS161, and on any given day, 5000 people are present on Stanford campus (including the 300 CS161 students). 10% of students enrolled in CS161 attend lectures in person, and a person on campus who isn't enrolled in CS161 has a 0.5% chance of wandering into a lecture anyway. If you see someone attending lecture in person, what is the probability they are enrolled, rounded to the nearest percent?

(a) 21% (b) 56% (c) 67% (d) 92%

2.8 Random permutation

Pick a uniformly random permutation of the digits 1 to n. What is the expected number of adjacent pairs of digits such that the first digit in the pair is less than the second? For example, in the permutation $(4, 1, 5, 2, 3)$, there are two such pairs: $1, 5$ and $2, 3$.

(a) $\log n$ (b) $2\sqrt{n}$ (c) $\frac{1}{2}(n-1)$ (d) $\frac{1}{2}n$

2.9 Independence

Answer true or false to the following questions about independence. Let A, B, C be events, and let X, Y be random variables.

1. If A and B are independent, then their complements A^c and B^c are also independent.
2. If A and B are independent, then they are also independent conditioned on any C.
3. If A and B are independent conditioned on any C, then they are independent.
4. If A and B are independent, and B and C are independent, then A and C are independent.
5. If A and B are independent, and B and C are independent, and A and C are independent, then A, B, and C are independent.

6. If X and Y are independent, then $P(X > Y) = P(Y > X)$.
7. If X and Y are independent, then $P(X > 10, Y < 10) = P(X > 10)P(Y < 10)$.
8. If X and Y are independent, then $\text{Var}(XY) = \text{Var}(X)\text{Var}(Y)$.
9. If X and Y are independent, then $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$.

3 Asymptotic Analysis

3.1

Determine if each function is asymptotically equivalent to n^2. All logarithms are base 2.

1. $2n^2$
2. $(n + 1)(n - 2)$
3. $2^{\sqrt{2n}}$
4. $\log(1 + 2^{n^2})$
5. $n^2 \log \log n$
6. $\frac{(n+1)^2(n-3)^2}{n^2+4}$
7. $\frac{n^3}{n+\log n}$
8. $\binom{2n+1}{2}$
9. $\sum_{k=1}^{\infty} \frac{n^k}{2^k \cdot k!}$

3.2

For each pair of functions, is $f(n) = O(g(n))$?

1. $f(n) = n^2,\ g(n) = n \log n$
2. $f(n) = 43n^2 + 228n + 91,\ g(n) = n^2$
3. $f(n) = n \log n,\ g(n) = n \log \log n$
4. $f(n) = \log(n^2 + 2n + 1),\ g(n) = \log n$
5. $f(n) = 3^n,\ g(n) = 2^n$
6. $f(n) = 2^{n^{1/2}},\ g(n) = n^3$
7. $f(n) = n,\ g(n) = 2^{\sqrt{\log n}}$
8. $f(n) = (\log n)^n,\ g(n) = n^{\log n}$